

Lincoln University Digital Thesis

Copyright Statement

The digital copy of this thesis is protected by the Copyright Act 1994 (New Zealand).

This thesis may be consulted by you, provided you comply with the provisions of the Act and the following conditions of use:

- you will use the copy only for the purposes of research or private study
- you will recognise the author's right to be identified as the author of the thesis and due acknowledgement will be made to the author where appropriate
- you will obtain the author's permission before publishing any material from the thesis.

ECONOMIC ASPECTS OF MARKET SEGMENTATION
WITHOUT SUPPLY CONTROL

A thesis
submitted in partial fulfilment
of the requirements for the degree
of
Doctor of Philosophy
in the
University of Canterbury

by
Sandra Martin

Lincoln College

1986

ABSTRACT

of a thesis submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy in the University of Canterbury, 1986.

Economic Aspects of Market Segmentation without Supply Control

by Sandra Martin

In recent years, marketing agencies operating on behalf of New Zealand agricultural industries in export markets have placed increasing emphasis on market segmentation policies as a means of achieving producer objectives. However, many of the prescriptions for segmentation have been developed within the context of monopolistic or oligopolistic industries. Firms operating under these conditions have no or few competitors and have the ability to control output to profit maximizing levels. However, typically structured agricultural industries do not have these features.

In this study, an economic model of market segmentation without supply control was constructed. The model incorporated the optimal allocation of industry output to any number of market segments, and included aspects of promotion given this optimal pricing behaviour.

From the model, it was determined that market segmentation outcomes were influenced by a number of variables. Producer returns were high when the price elasticities of demand and supply were low, and the

divergence between demand elasticities in individual market segments was large. It was also noted that these demand elasticities are themselves influenced by a further set of factors, including the market share held in a particular market segment, the extent of product differentiation and competitive supply responses. Analysis of the promotion component of the model suggested that promotion activity should be carefully targeted, with more effort being directed towards less price elastic market segments.

An application of the model to the export of New Zealand sheepmeats indicated that gains from market segmentation activity were high in the short-term. However, they were heavily eroded in the long-term, largely as a result of competitive supply responses.

KEYWORDS: Agricultural marketing, agricultural marketing institutions, market segmentation, price discrimination, optimal pricing, optimal promotion, supply response.

ACKNOWLEDGEMENTS

I would like to thank my supervisor Professor Tony Zwart for his excellent guidance and for his support during the preparation of this thesis. I am also grateful for the assistance given to me by Professor Leslie Young in solving mathematical aspects of the promotion issue analysed in Section 5.3.3 of the study.

I would like to acknowledge the financial support given to me by various sponsors over the period of this research. These include the Reserve Bank of New Zealand for the award of a Research Fellowship, the Department of Trade and Industry who also supported my work, and Lincoln College for the award of the Sir John Ormond Fellowship and through the generous provision which was made for me to complete my research when I commenced my employment in 1985.

I am also grateful for the support given to me by my Head of Department, Dr Alistair McArthur, and by staff of the Department of Agricultural Economics and Marketing and the Agricultural Economics Research Unit. In particular, I wish to thank the Director of the Agricultural Economics Research Unit, Dr Ralph Lattimore, for his constant encouragement.

I would also like to thank Wendy Lamb for her patient and accurate typing of this thesis.

Finally, I would like to express my gratitude to my husband Dick and daughter Linda for their patience, support and encouragement.

CONTENTS

	PAGE
ABSTRACT	
ACKNOWLEDGEMENTS	
LIST OF TABLES	(ix)
LIST OF FIGURES	(x)
CHAPTER	
1.0 INTRODUCTION	1
1.1 Statutory Intervention in the Marketing of New Zealand's Major Agricultural Exports	1
1.2 Study Rationale	3
1.3 Objectives of the Study	7
1.4 Organization of the Study	8
2.0 THE INSTITUTIONAL STRUCTURE IN NEW ZEALAND'S MAJOR AGRICULTURAL EXPORTING INDUSTRIES	10
2.1 Introduction	10
2.2 Alternative Frameworks for Evaluation	11
2.2.1 Analysis of Legislation ..	11
2.2.2 Institutional Type	12
2.2.3 Institutional Behaviour ..	13
2.2.4 An Evolutionary Approach ..	15
2.2.5 Source Material for an Evolutionary Approach	19
2.3 The Evolution of Statutory Structures the Dairy Industry	20
2.3.1 The Market Environment and Statutory Institutional Activities	20
2.3.2 Economic Policies of Statutory Institutions	29
2.3.3 Objectives of Statutory Institutions	31

CHAPTER	Contents (continued)	PAGE
2.3.4	Effective Powers of Statutory Institutions	34
2.3.5	Summary of Objectives, Effective Powers and Economic Policies of Statutory Institutions	36
2.4	The Evolution of Statutory Structures the Meat Industry	37
2.4.1	The Market Environment and Statutory Institutional Activities	37
2.4.2	Economic Policies of Statutory Institutions	47
2.4.3	Objectives of Statutory Institutions	49
2.4.4	Effective Powers of Statutory Institutions	52
2.4.5	Summary of Objectives, Effective Powers and Economic Policies of Statutory Institutions	54
2.5	The Evolution of Statutory Structures in the Wool Industry	56
2.5.1	The Market Environment and Statutory Institutional Activities	56
2.5.2	Economic Policies of Statutory Institutions	63
2.5.3	Objectives of Statutory Institutions	65
2.5.4	Effective Powers of Statutory Institutions	68
2.5.5	Summary of Objectives, Effective Powers and Economic Policies of Statutory Institutions	69
2.6	The Evolution of Statutory Structures in the Pipfruit Industry	71
2.6.1	The Market Environment and Statutory Institutional Activities	71

Contents (continued)		PAGE
CHAPTER		
	2.6.2 Economic Policies of Statutory Institutions	76
	2.6.3 Objectives of Statutory Institutions	77
	2.6.4 Effective Powers of Statutory Institutions	79
	2.6.5 Summary of Objectives, Effective Powers and Economic Policies of Statutory Institutions	80
2.7	The Evolution of Statutory Structures in the Kiwifruit Industry	81
	2.7.1 The Market Environment and Statutory Institutional Activities	81
	2.7.2 Economic Policies of Kiwifruit Agencies	87
	2.7.3 Objectives of Kiwifruit Agencies	87
	2.7.4 Effective Powers of Kiwifruit Agencies	89
2.8	Conclusions	89
	2.8.1 The Evolution of Market Segmentation Policies	89
	2.8.2 Representative Objectives of Statutory Agencies	92
	2.8.3 Representative Effective Powers of Statutory Agencies	94
3.0	A FRAMEWORK FOR EVALUATING MARKET SEGMENTATION POLICIES	97
3.1	Introduction	97
3.2	An Economic Model of Market Segmentation	98
	3.2.1 Market Segmentation and Marketing Management	98
	3.2.2 Price Discrimination Defined	100

Contents (continued)		
CHAPTER		PAGE
	3.2.3 The Third-Degree Monopolistic Price Discrimination Model ..	102
	3.3 Evaluation Criteria	105
	3.4 The Concept of Producer Surplus ..	106
	3.5 Review of the Literature	109
	3.5.1 Introduction	109
	3.5.2 Own Supply Response to Discriminatory Pricing	110
	3.5.3 Competitive Supply Response to Discriminatory Pricing	115
	3.5.4 Promotion Effects	117
	3.6 Conclusion	123
4.0	A THEORETICAL MODEL OF PRICE DISCRIMINATION WITHOUT SUPPLY CONTROL	125
	4.1 Introduction	125
	4.2 A Competitive Pricing Model with Multiple Market Segments	126
	4.3 Maximization of Returns under Price Discrimination without Supply Control	130
	4.4 A Generalized Model of Price Discrimination	134
	4.4.1 Optimal Quantities and Prices in Market Segments ..	134
	4.4.2 Revenue Gains from Price Discrimination	137
	4.5 An Own Supply Response to Discriminatory Pricing	144
	4.6 A Competitive Supply Response to Discriminatory Pricing	148
	4.7 Returns to Producers from Price Discrimination with Supply Response	154

Contents (continued)		
CHAPTER		PAGE
	4.7.1 Optimal Quantities and Prices in Individual Market Segments with Supply Response	154
	4.7.2 Revenue Gains from Price Discrimination with Supply Response	156
	4.7.3 Producer Surplus Gains from Price Discrimination with Supply Response	162
	4.8 Conclusion	164
5.0	ASPECTS OF PROMOTION WITH SUPPLY RESPONSE	168
	5.1 Introduction	168
	5.2 Conventional Models of Optimal Promotion used for Agricultural Industries	169
	5.3 Extensions to the Nerlove-Waugh Theorem	176
	5.3.1 Introduction	176
	5.3.2 Optimal Advertising in Multiple Markets under Competitive Pricing	176
	5.3.3 Optimal Advertising under Discriminatory Pricing with a Competitive Supply Response	187
	5.3.4 Summary of Nerlove-Waugh Extensions	195
	5.4 The Influence of Generic Promotion on Producer Returns	198
	5.4.1 Introduction	198
	5.4.2 A Competitive Pricing Model with Generic Promotion	202
	5.4.3 A Discriminatory Pricing Model with Generic Promotion	208
	5.4.4 Conclusion	213

Contents (continued)		
CHAPTER		PAGE
5.5	The Effect of Product Differentiation on Producer Surplus under Discriminatory Pricing	214
5.6	Summary and Conclusions	216
6.0	THE RELATIONSHIP BETWEEN MARKET CHARACTERISTICS AND THE OUTCOMES OF MARKET SEGMENTATION POLICIES	221
6.1	Introduction	221
6.2	Basic Model Features	222
6.2.1	Variables Influencing Segmentation Outcomes	222
6.2.2	A Base Model	226
6.3	Variables Influencing Short-Run Outcomes	229
6.3.1	The Competitive Equilibrium Position	229
6.3.2	Price Elasticities of Demand	229
6.3.3	Shares of Output Allocated to each Segment	235
6.3.4	Conclusion	240
6.4	Variables Influencing Long-Run Outcomes	240
6.4.1	The Aggregate Price Elasticity of Demand	240
6.4.2	The Price Elasticity of Supply	242
6.4.3	Demand Shifts from Generic Promotion	245
6.4.4	Conclusion	251
6.5	Factors Influencing Specific Long-Run Variables	252
6.6	The Influence of Market Segmentation Policies on Producer Price Variability	257

Contents (continued)		PAGE
CHAPTER		
6.7	An Application to the New Zealand Sheepmeats Industry	260
7.0	CONCLUSIONS	269
7.1	Introduction	269
7.2	Summary	269
7.2.1	Outline of the Study	269
7.2.2	A Summary of Results	272
7.2.3	Qualification of the Model and its Results	275
7.3	A Review of Results with Respect to the Literature	277
7.3.1	Introduction	277
7.3.2	Short-Run Considerations	277
7.3.3	Own Supply Response	278
7.3.4	Competitive Supply Response	280
7.3.5	Promotion	281
7.3.6	The Influence on Producer Price Variability	282
7.4	Directions for Further Research	282
7.5	Implications for Marketing Agencies	285
REFERENCES	292
APPENDICES	305
1	Proofs to Equations in Chapter 4	305
2	Proofs to Selected Equations in Chapter 5	344
3	Proofs to Selected Equations in Chapter 6	365

APPENDICES	Contents (continued)	PAGE
4	Reprint of Martin, S. K., Young, L. and Zwart, A. C. Optimal Pricing and Promotion for Agricultural Marketing Agencies. <u>Research</u> <u>Report No. 177</u> . Agricultural Economics Research Unit, Lincoln College. 1986. 21p.	367

LIST OF TABLES

TABLE	TITLE	PAGE
6.1	Basic Variables which Influence the Outcomes of Market Segmentation Policies	227
6.2	Factors Influencing the Magnitude of Long-Run Variables	228
6.3	Base Model Parameters	230
6.4	The Influence of Varying Levels of n_1 on Short-Run Producer Returns	232
6.5	The Influence of Varying Levels of n_2 on Short-Run Producer Returns	234
6.6	The Influence of Varying Levels of n_1 and n_2 , with Constant Divergence, $(n_2 - n_1)$, on Short-Run Producer Returns	236
6.7	The Influence of Varying Levels of n_1 and n_2 , with Constant n_a , on Short- Run Producer Returns	237
6.8	The Influence of Varying Proportions of Output Allocated to Individual Segments, s_1 and s_2 , on Short-Run Producer Returns	239
6.9	The Influence of the Aggregate Demand Elasticity, n_a , on Long-Run Producer Returns	243
6.10	The Influence of the Supply Elasticity, e , on Long-Run Producer Returns	246
6.11	The Influence of Generic Promotion on Long-Run Producer Returns	248
6.12	The Influence of Product- Differentiating Promotion under Discriminatory Pricing	256
6.13	Data Used in Sheepmeat Market Analysis	263
6.14	Market Segmentation in the Sheepmeat Market - Short-Run	265
6.15	Market Segmentation in the Sheepmeat Market - Long-Run	266

LIST OF FIGURES

FIGURE	TITLE	PAGE
2.1	Spectrum of Marketing Authorities' Functions	14
2.2	Framework for Evaluating Institutional Evolution	18
2.3	The Evolution of Statutory Marketing in New Zealand's Dairy Industry ..	32
2.4	The Evolution of Statutory Marketing New Zealand's Meat Industry	50
2.5	The Evolution of Statutory Marketing in New Zealand's Wool Industry ..	66
2.6	The Evolution of Statutory Marketing in New Zealand's Pipfruit Industry	78
2.7	Statutory Marketing in New Zealand's Kiwifruit Industry	88
4.1	Revenue Gains from Price Discrimination	141
4.2	Own Supply Response to Discriminatory Pricing	146
4.3	Producer Returns from Discriminatory Pricing	158
5.1	Models Showing Optimal Advertising Intensities under Different Market Conditions	196
5.2	Producer Surplus Gains from Generic Promotion under Competitive Pricing	206
5.3	Producer Surplus Gains from Generic Promotion under Discriminatory Pricing	212

CHAPTER 1

INTRODUCTION

1.1 STATUTORY INTERVENTION IN THE MARKETING OF NEW ZEALAND'S MAJOR AGRICULTURAL EXPORTS

Traditionally, New Zealand's agricultural exports have been dominated by three industries, these being meat, dairy and wool. In addition, exports of horticultural products have increased in significance in recent years, and, of these products, kiwifruit and pipfruit are the most important.

In 1984, these five industries returned \$4427 million (f.o.b. value) in export receipts, which was 75 per cent of the total value of agricultural based exports, and 51 percent of the total value of all New Zealand exports. Of the export revenue from these five products, meat contributed 39 percent, dairy products 31 percent, wool 25 percent, kiwifruit 2 percent and pipfruit 3 percent (Department of Statistics, 1984). Consequently, export marketing arrangements in these industries impinge on a large proportion of New Zealand's export receipts, and collective marketing activities by their industry agencies are of some national significance.

Each of these industries is characterized by an institution, created under statutory authority, which is involved, to a greater or lesser extent, in the export marketing of these products. These institutions are the

New Zealand Meat Producers' Board, the New Zealand Dairy Board, the New Zealand Wool Board, the New Zealand Kiwifruit Authority, and the New Zealand Apple and Pear Marketing Board.

In general, each of these institutions has statutory authority to significantly control the export marketing of their particular product, although they are not authorized to control the level of output produced. In practice they may not fully utilize these powers although this does occur in a majority of cases, with the New Zealand Dairy Board, the New Zealand Apple and Pear Marketing Board, and until recently the New Zealand Meat Producers' Board, exercising almost full control over the export marketing of their products. The New Zealand Kiwifruit Authority and the New Zealand Wool Board are involved in exporting activity to a lesser extent.

With the exception of the New Zealand Kiwifruit Authority, these institutions were created in the early 1920's. They were established following a period of depressed and unstable prices, and were given wide-ranging powers to control the flow of exports. Over the years, the types of economic activity in which they have engaged appears to have changed in response to changes in the economic and political environments in their industries, and casual observation suggests that current policy emphasis is being placed on the management of marketing activity in specific markets.

The New Zealand Kiwifruit Authority arose much more recently in 1977. Once again, its creation was a response

to a perceived unfavourable price situation. In this case, it was feared that prices would be depressed by the entry into the industry of new suppliers and exporters marketing low quality product. Therefore, it was given power to control some exporting activity.

1.2 STUDY RATIONALE

In the current market environment facing many export commodities, agricultural marketing agencies are being urged to place increasing emphasis on marketing management strategies as a means of increasing returns to producers. In many developing industries, such as the newer horticultural ones, this has led to pressures to create new institutions modelled on those in established industries, which are perceived as being able to implement such strategies.

Although such strategies are being emphasized, the actual extent to which they have been adopted by marketing agencies operating in agricultural exporting industries is unclear. Nor is it obvious what benefits accrue from such policies. However, casual observation would suggest that a crucial element in any marketing management strategy is the ability to exploit opportunities which might arise from segmenting the export market. This could occur if New Zealand's products face downward sloping demand curves in any or all of such market segments. This would allow an institution to exploit a monopolistic advantage in particular markets which would not be possible with uncontrolled competitive marketing arrangements.

If market segmentation activities are to be adopted by marketing agencies, then it implies that they must have the power to control the pricing and destination of their products on export markets. Some agencies may already be using their powers to this end, whereas others have such powers, but may not necessarily be utilizing them. If such activities are seen as essential in this latter type of industry, and in developing industries with no statutory marketing authority in place, then considerable industry restructuring may be involved.

A study of the potential benefits to producers from market segmentation activities by agencies operating on their behalf would appear to be justified on the following grounds.

(a) Benefits to Statutory Authorities

Marketing authorities already in existence attempt to fulfil a set of objectives in accordance with their statutory obligations. It is generally thought that such agencies have three fundamental long-term objectives. These are to increase returns to producers, to stabilize these returns, and to ensure an acceptable degree of equity in their treatment of producers. However, it is not known which of these objectives is most important, nor whether they vary between industries. When such institutions are considering the introduction or assessment of market segmentation strategies, they will be interested in the extent to which such an activity will assist in achieving specific producer objectives.

However, many of the managerial prescriptions for market segmentation which result from a marketing management perspective have been derived within the context of monopolistic or oligopolistic market structures. Unfortunately, some of the product, market and organizational features of these industries tend to differ from those which are typical of agricultural industries. In particular, suppliers in imperfectly competitive markets will tend to adjust output to profit maximizing levels in response to their specific demand conditions. The many suppliers in an agricultural market, however, will respond in a competitive manner when adjusting output in response to the marketing arrangements of an institution operating on their collective behalf.

Therefore, it is not obvious whether an unmodified application of these marketing principles will result in benefits to agricultural suppliers which are of a similar magnitude to those which might be expected in industries which are less competitively structured. Under these circumstances, marketing organizations may find it difficult to assess the extent to which market segmentation activities will assist them in achieving their institutional objectives.

(b) Cost of Industry Restructuring

The second justification for a study of the potential benefits from market segmentation in agricultural exporting industries revolves around the cost of industry restructuring which would arise if such a policy was introduced in an industry where the pricing and

destination of output is not effectively controlled under existing marketing arrangements. Such a cost could include the following components.

Firstly, administrative costs may be incurred by Government in both the setting up of statutory marketing authorities, and in counterpart liaison and monitoring activities by Government Departments.

Less obvious indirect costs may result from inefficiencies in resource allocation which could occur through interference in an otherwise competitively functioning marketing channel. For example, a statutory marketing organization might institute a controlled export marketing programme by commissioning licenced exporters to market specified volumes of product in specific markets, thereby creating rents associated with these licensing opportunities. This can lead to considerable private resources being diverted from productive activity to unproductive rent-seeking behaviour.

In addition, once statutory marketing is introduced it may crowd out private marketing activity, and can tend to become entrenched. Hence over time the competitive benchmark to marketing activity is lost, and it becomes very difficult to assess, ex post, the economic efficiency of such statutory marketing structures.

Therefore, social costs may be incurred if it is necessary to introduce statutory marketing to implement market segmentation strategies. Consequently, policy-makers require some knowledge of the potential magnitude of benefits to producers which would arise from

such an activity. This would allow them to assess whether the granting of property rights to facilitate statutory marketing justifies the direct tax expenditure which will be incurred, and the inefficiencies which may result.

1.3 OBJECTIVES OF THE STUDY

Given the rationale established above, the general aim of this study is to investigate the nature of benefits to producers from market segmentation through controlled export marketing within the market and organizational context of a typically structured agricultural industry. This would allow statutory marketing institutions to assess the extent to which such strategies assist in achieving their objectives. It also enables policy-makers to determine whether statutory intervention for this purpose in an otherwise competitively functioning marketing channel justifies the cost of so doing.

To this end, the study has the following four objectives.

- (1) The first objective is to establish whether market segmentation is a significant policy of statutory agencies operating in New Zealand's major agricultural exporting industries.
- (2) The second objective is to determine what organizational features may be considered typical in these exporting industries.
- (3) The third objective is to develop a theoretical model of market segmentation which incorporates the appropriate organizational features.

- (4) The fourth, and most important, objective is to determine the market conditions under which market segmentation activities by statutory marketing agencies operating in typically structured agricultural industries are likely to be beneficial in terms of producer objectives.

1.4 ORGANIZATION OF THE STUDY

Chapter Two outlines the organizational features of New Zealand's five major agricultural exporting industries, and establishes the relative importance of market segmentation policies in these industries. This is done by examining the evolution of the statutory marketing institutions currently in operation, thereby allowing the development in their policies to be studied, and their objectives to be implicitly determined. In addition, their current powers with respect to export marketing are outlined, and the extent to which they utilize these powers is assessed. This Chapter fulfills the first and second objectives of this study.

Chapter Three establishes the theoretical framework necessary for constructing a model of market segmentation. The pertinent literature is reviewed, an economic framework is established, and the advantages and disadvantages of this framework are discussed. In Chapter Four, a basic theoretical model of market segmentation without supply control is derived, and the factors influencing the magnitude of gains from this strategy are identified. This basic model optimizes the price and the

volume of product in each market segment. Chapter Five then considers the implication of specific promotional activities for the pricing strategy considered in this basic model. The third study objective is met in these three Chapters.

In Chapter Six the model is quantified using simulated sets of market characteristics. It is then utilized, both analytically and numerically, to determine the influence of alternatives market characteristics on producer objectives, thereby fulfilling the final study objective.

In conclusion, Chapter Seven summarizes and discusses the results of the study, notes appropriate caveats on their use, and discusses their implications for agricultural and horticultural marketing institutions which are practising or contemplating market segmentation activities.

CHAPTER TWO

THE INSTITUTIONAL STRUCTURE IN NEW ZEALAND'S MAJOR AGRICULTURAL EXPORTING INDUSTRIES

2.1 INTRODUCTION

This Chapter reviews the statutory economic policies and organizational structure in New Zealand's major agricultural exporting industries. On the basis of this Review, the relative significance of market segmentation policies by statutory agencies is established. A representative set of objectives for a typically structured institution operating in the export sector is then developed, and the analysis is used to derive a spectrum of powers which typify such an institution. These institutional features will then be utilised in Chapter Four, where a market segmentation model is developed for a representative agricultural exporting industry.

The remainder of the Chapter is structured as follows. Section 2.2 considers alternative frameworks for evaluating institutional objectives and effective powers. In Sections 2.3 to 2.7, the economic policies, agency objectives, and effective powers of institutions operating in the five major agricultural and horticultural exporting industries are derived. By way of conclusion, Section 2.8 discusses the relative importance of market segmentation policies, and a representative set of objectives and effective powers are determined by synthesizing the

information collated and analyzed for individual industries.

2.2 ALTERNATIVE FRAMEWORKS FOR EVALUATION

2.2.1 Analysis of Legislation

At first glance, it might seem obvious that a standardized set of institutional objectives and powers could be distilled from the enabling legislation currently binding the appropriate institutions. However, a closer perusal of the relevant Parliamentary Acts and Regulations suggests that such an approach is too simplistic for the following reasons.

In the first place, institutional objectives might not be legislatively stated explicitly (Dairy Board Act 1961; Apple and Pear Marketing Act 1971; Kiwifruit Marketing Licensing Regulations 1977). Alternatively, they may be too vague to be functionally useful. For example, institutions might be created so that economic welfare will be promoted (Meat Export Control Act 1921-22), or such agencies might be charged with obtaining the best possible long-term returns for producers (Wool Industry Act 1977).

A further complication arises with this approach because institutions may not fully utilize their powers to control conduct in the export sector of an industry, despite being vested with such authority. This appears to be the case with the New Zealand Wool Board in particular. In these circumstances, a perusal of existing legislation might not capture political inhibitions to institutional

behaviour, and therefore, may give a misleading impression of the effective, as opposed to the actual, powers of such agencies.

2.2.2 Institutional Type

A popular taxonomy in the literature classifies institutions according to type, where this appears to be defined by legislative status. Therefore, research may be concentrated on a particular institutional type such as co-operatives, marketing agreements, commodity commissions, licensing authorities, marketing orders and boards (Hoos, 1979; Warley, 1967; Veeman, 1972; LeVay, 1983). Alternatively, comparisons may be made between institutional types (Davies, 1960; Cohen, 1961; Morley, 1967; Campbell, 1973; Rae, 1979). The objectives and behaviour of such organizations tend to be implied by their institutional type. For example, it is generally accepted that the objectives of Boards are to enhance producer returns, to ensure equity among producers, and in some cases, to stabilize prices (Currie and Hoos, 1979; Schmitz and McCalla, 1979; Veeman, 1979; Melamed, 1979), although it is acknowledged that individual Boards may differ in this respect (deVos, 1979; Blandford, 1979).

However, there are some deficiencies with this system of classification when using it to determine agency objectives and effective powers. For example, some organizational types, such as boards and marketing orders, are essentially national variants of the same institutional structure (Hoos, 1979; Warley, 1967).

More importantly, some agencies such as Boards can incorporate a wide range of behaviour, with corresponding implied differences in powers and acceptable methods of achieving objectives (Currie and Hoos, 1979). For instance, Morley (1967) has classified Boards into six broad types. These are advisory and promotional, non-trading regulatory, non-trading price stabilization, trading price stabilization, monopoly export boards and domestic monopoly boards. It is obvious, therefore, that a wide range of behaviour is encompassed in this one organizational type.

2.2.3 Institutional Behaviour

An alternative method of classification which tends to overcome the above deficiencies, is to order marketing institutions according to their functions. Balderstone et al (1982) in their study of statutory marketing authorities in Australia, use this approach to indicate a spectrum of involvement by agencies in the marketing process. Figure 2.1 reproduces their spectrum, which ranges from minimum market intervention with grading to maximum market intervention with supply controls.

While a system such as this orders institutions according to market behaviour, such behaviour tends to be a mixture of market activities such as storage and transport, and economic policies such as price influencing. However, storage activities could conceivably form part of a price influencing policy. Therefore, such a classification system may not be

Minimum market intervention

Maximum market intervention

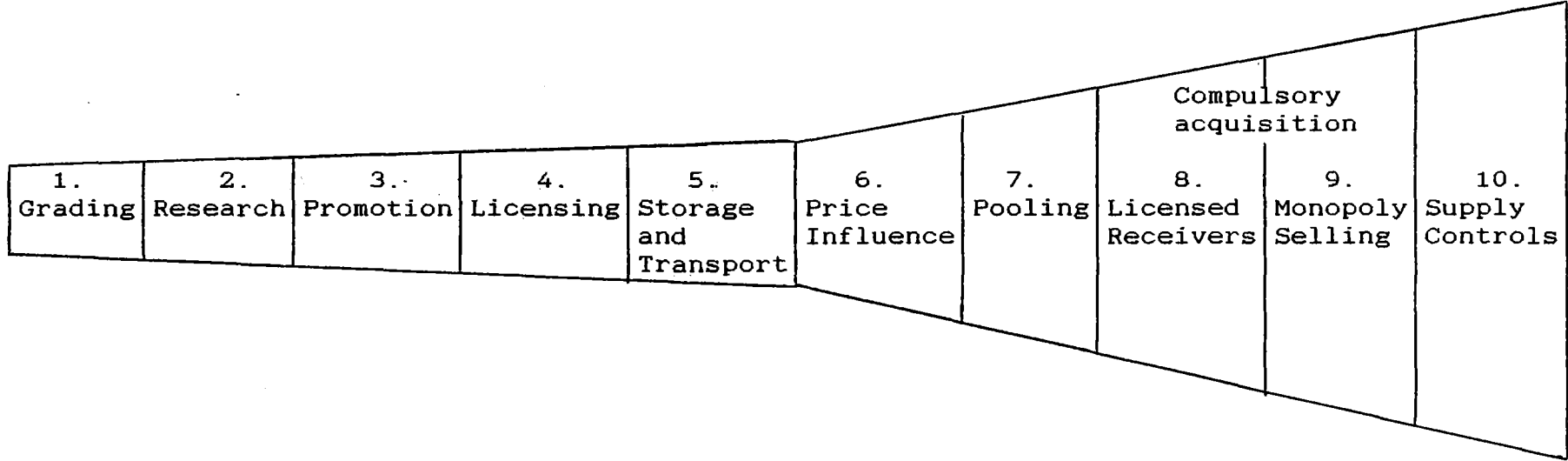


FIGURE 2.1: SPECTRUM OF MARKETING AUTHORITIES' FUNCTIONS

Source: Balderstone et al (1982)

mutually exclusive, since it incorporates activities of different orders. In addition, the objectives of institutions may not be obvious from institutional behaviour alone.

2.2.4 An Evolutionary Approach

A fourth alternative which has been used to determine, among other things, the fundamental objectives of marketing agencies is to analyze the evolutionary development of these organizations (Veeman, 1972; Campbell, 1973). Using this approach, market conditions which lead to the establishment of these institutions are examined, and historical changes in their activities analyzed. This makes it possible to infer their long-term objectives from their responses to changing market conditions over time.

This method of analysis does not exclude the three approaches already discussed, since it tends to incorporate elements of all of them, but with the addition of a temporal dimension. Therefore, as institutions evolve, their empowering legislation must be analyzed, their activities commented on, and their institutional type noted.

There are advantages to using this method for the purposes of this study. Firstly, the problem of assessing institutional objectives can be directly addressed using this approach. Secondly, through an analysis of organisational behaviour over time, the extent to which such institutions appear to be constrained by political

rather than legislative considerations can be determined. This would allow an assessment to be made of the effective powers of such agencies, as opposed to their actual legislative powers. In addition, the evolution of statutory economic policies over time can be traced, thereby allowing the relative significance of market segmentation policies to be assessed.

However, a potential problem which could be encountered with such an approach for the purposes of this study, is the lack of analytical precision which may result from a purely descriptive discussion of evolutionary trends in agency activities in the relevant industries. For example, Veeman (1972) analyzed the activities of statutory agencies in the New Zealand dairy, meat and wool industries over time. However, she was not concerned with actually deducing institutional objectives, as this study is, but with evaluating whether such activities achieved the generally accepted objectives of Boards noted in Section 2.2.2. She qualifies her analysis by stating that boards have a variety of objectives which differ from board to board, and that board objectives might not necessarily be consistent.

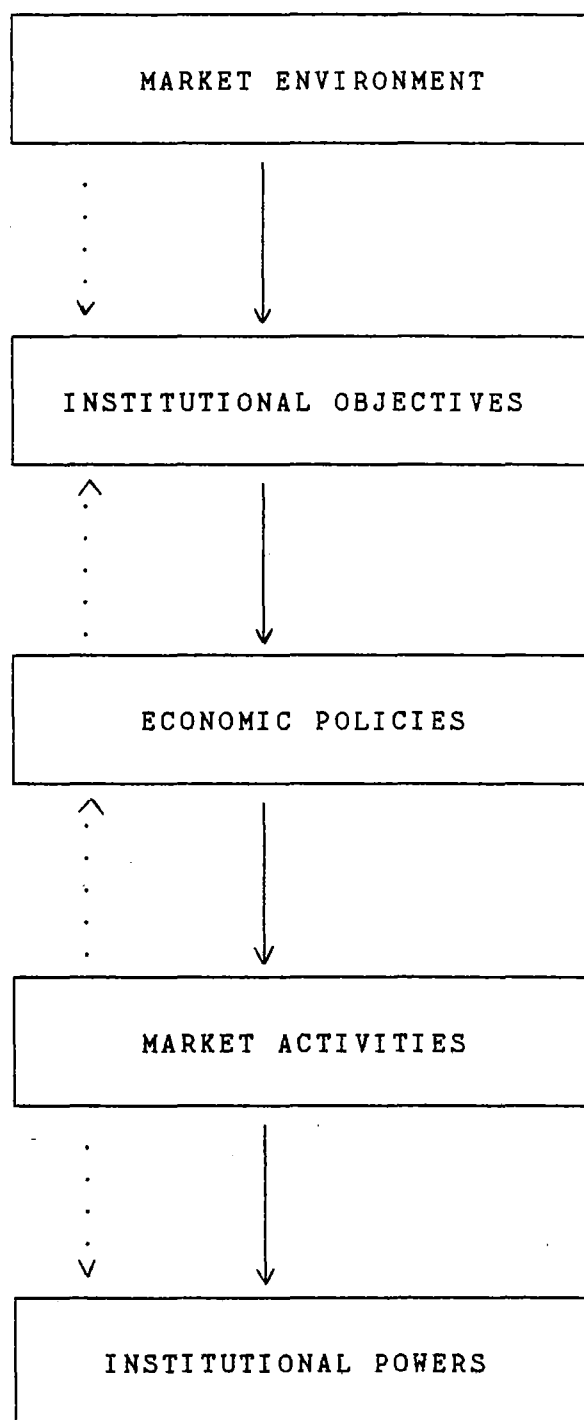
For the purpose of this study, Veeman's generalized approach is not suitable, since it is important to establish the objectives of individual agencies, the stability of these objectives over time, the relative importance of specific objectives, and the robustness of a derived set of representative objectives with respect to specific industries. Therefore, in order to achieve this,

a framework for discussion has been derived, which is diagrammatically represented in Figure 2.2.

This framework envisages that the market environment in which an industry operates will determine the primary objectives of a statutory institution. These objectives, in turn, determine the broad economic policies which such an institution would undertake. Such policies might include, among others, demand or supply shifting behaviour, stabilization activities, supply restriction, and market segmentation. These broad policies would, in turn, require the use of certain market activities by the organization. Examples of these could include promotion, research, storage, buffer price activities, quota fixing and allocation, and distribution of product to alternative destinations. Hence, a distinction is made between broad economic policies and the actual market activities in which an institution is involved, thus avoiding the problems associated with comparing functions of a different order noted in Section 2.2.3. This set of market activities, in turn, determines the nature of the institution required, and demonstrates its effective powers. In Figure 2.2, the envisaged link from the market environment through to the institutional powers is shown by the solid set of arrows.

In determining the objectives and effective powers of such institutions, analysis is focussed on observable features such as the market activities of institutions, and the market environment in which they operate. Therefore, an analysis of market activities gives an

FIGURE 2.2
FRAMEWORK FOR EVALUATING INSTITUTIONAL EVOLUTION



indication of the effective powers of an institution and the economic policies they are attempting to carry out. In turn, these economic policies, when considered with the market environment, suggest the fundamental objectives of the institution. The dotted arrows in Figure 2.2 indicate this deductive process. Note that, unlike Veeman (1972), the objective of this framework is not to assess whether agency activities actually achieve specified objectives, but to determine what implied objectives economic policies may be designed to achieve.

In Section 2.3, this framework is applied to each of the five industries to be analyzed. The period over which each of these industries has been in operation is divided into discrete time intervals, each of which is characterized by a particular market environment. For each time period, an attempt is then made to deduce economic policies, institutional objectives and effective powers from the activities of organizations under the ruling market environment. Due to limitations imposed by accessible historical documentation, some of this analysis is somewhat sketchy. However, despite this, it is adequate for deducing the long-run economic policies, institutional objectives and effective powers of current institutions.

2.2.5 Source Material For An Evolutionary Approach

Previous researchers (Veeman, 1972; Woods, 1981; Yerex and Haines, 1983) have conducted primary research into the

historical evolution of statutory marketing in New Zealand's major agricultural exporting industries. The objective of this brief synopsis is to comment on economic policies, institutional objectives and powers over time, rather than to derive base information. Therefore, material is synthesized from these and other secondary sources and not primary ones, except where the use of primary sources is necessary to update or elucidate material in these earlier studies. Such primary sources include the Annual Reports since 1981 of the New Zealand Dairy Board, the New Zealand Meat Producers' Board, the New Zealand Wool Board, the New Zealand Apple and Pear Marketing Board and the Annual Reports of the New Zealand Kiwifruit Authority since its inception in 1978.

2.3 THE EVOLUTION OF STATUTORY STRUCTURES IN THE DAIRY INDUSTRY

2.3.1 The Market Environment and Statutory Institutional Activities

2.3.1.1 Prior to 1921

Statutory involvement in the dairy industry began in the last century with the 1894 Dairy Industry Act, which provided for the grading of butter and cheese to be undertaken by government personnel.

Between 1894 and 1914, a free enterprise system was in operation, under which New Zealand dairy companies sold export produce directly to Great Britain, either on open consignment, or on an f.o.b. basis at an agreed price.

Assertions of monopsonistic exploitation by British merchants led to various unsuccessful attempts to organize collective marketing schemes prior to 1914. In addition, problems of co-ordination appeared to arise from variations in company payouts, which led to suppliers re-diverting output to temporary payout leaders, thereby causing difficulties for factories in scheduling production, and in increased transport costs.

During the First World War, an Imperial Commandeer of export dairy produce was in operation. Under this arrangement, the United Kingdom bulk purchased all New Zealand butter and cheese which was surplus to domestic requirements. Prices for the contract were negotiated annually by the two Governments, and, over the period of the Commandeer, tended to be relatively high.

Therefore, prior to 1914, the market environment under free enterprise was perceived by producers to have monopsonistic elements, and seemed to be characterized by an element of instability which led to co-ordination problems. On the other hand, high stable prices were a feature of the Commandeer.

2.3.1.2 1921 to 1934

The return to free enterprise in 1921 coincided with a period of low prices, which largely resulted from oversupply, since the UK market was the only major market available to several dairy exporters. This was in contrast to the Commandeer, and an association seems to have been made between relatively high, stable prices,

and 'central control' in the industry, which appears to have led to demands for centralized marketing legislation.

Such pressure resulted in the 1923 Dairy Produce Export Control Act, which allowed for the establishment of the Dairy Produce Control Board in 1924. This Act made provision for price fixing, the regulation of shipments and insurance of produce in transit.

The Dairy Produce Control Board commenced operation and, in 1926, assumed the power of complete ownership of all export produce, which was then sold only through selected agents at minimum prices set by the Board.

However, these arrangements met with opposition from a number of quarters. Many producers were uneasy about the principle of acquisition and resented seasonal price pooling arrangements, while British merchants feared monopolistic exploitation. In addition, because of a weak market situation, the Board was forced to successively lower its minimum prices.

As a consequence of these events, the Board abandoned its policy of 'absolute control' after one year, and, in 1927, adopted a policy of 'limited intervention' within a free-market system. Board activities were restricted to advertising in the United Kingdom market, arranging shipping and freight contracts, allocating shipping space so as to spread shipments throughout the season, and supporting, through grants, scientific research into the dairy industry.

2.3.1.3 1934 to 1945

The Depression marked a period of low export prices and market access problems. As a result, a Royal Commission of Enquiry into the Dairy Industry was set up, which resulted in the 1934 Agriculture (Emergency Powers) Act. The aim of this Act was to regulate the production and marketing of agricultural products, and to co-ordinate the activities of existing Boards. In addition, complete control of all dairy product marketing was vested in the New Zealand Dairy Board, which was the renamed Dairy Produce Control Board. By utilizing these powers, the Board intended to develop minimum price and group marketing schemes for export selling.

However, this proposed variant of the Board was largely inoperative as a result of the 1936 change of Government. As promised in its election manifesto, the Labour Government repealed the 1934 Act, and replaced it with the 1936 Primary Products Marketing Act. This provided for the establishment of a Primary Products Marketing Department, which had power to acquire all dairy produce and to fix an annual guaranteed price for such produce. This price was to be based on a number of factors. These were previous prices, costs involved in production and marketing, costs of administering the Act, general living standards in the dairy industry relative to those in other sectors, and the necessity in the public interest of maintaining the efficiency and stability of the dairy industry.

The Primary Products Marketing Department took over the storage, shipping, insurance, advertising and internal marketing functions of the Board, which was then left with the role of developing industry unity and co-operation.

However, in the following years, dairy industry representatives grew increasingly unhappy with these bureaucratic marketing arrangements. In particular, concern was expressed about the guaranteed price scheme, since it was felt that prices were being determined on the basis of current market realizations rather than on the cost of production criteria allowed for in the legislation. This dissatisfaction continued until the Second World War, which quelled disagreements with Government over guaranteed price levels to some degree.

During the Second World War, all exported dairy produce was sold under bulk contract agreements by the Government to the United Kingdom Government. During the War, Government emphasis was on macro-economic problems such as inflation. As part of their scheme to restrain internal prices, stabilization accounts were established for meat and dairy produce, which allowed for prices paid to farmers to be divorced from those received for exports. Allowances for increased costs incorporated into the guaranteed price were subsequently charged to the Dairy Industry Stabilization Account.

The initial guaranteed price in 1936 had been set high relative to the market price, but was essentially frozen for a number of years afterwards, during which time the market price had risen above the guaranteed price.

This led to a build-up of reserves in the Stabilization Account.

2.3.1.4 1945 to 1961

With the conclusion of the Second World War, farmer resentment over the price-determining process again built up. Producers were unhappy at the perceived gains made by organized labour relative to the farming community. In addition, it was felt that the Government had not driven as hard a bargain with the British Ministry of Supply as it might have done, since higher prices had been paid to other suppliers. Producers also resented the freezing of withheld export receipts in the farm stabilization accounts.

As a consequence of this intensive dairy industry pressure, the Dairy Products Marketing Commission Act was passed in 1947, which allowed for the formation of the Dairy Products Marketing Commission. The Dairy Board continued in its role of developing industry unity and co-operation, while the new Commission essentially took over the functions of the Primary Products Marketing Department. To alleviate producer dissatisfaction, the cost of production criterion was given increased weighting.

The bulk purchase agreement continued with the United Kingdom until 1954. During this period, industry reserves continued to build up. In 1952, the Government, the Board and the Commission held discussions on the guaranteed price procedures, and on the accumulated industry funds.

It was agreed by all parties that the purpose of the guaranteed price scheme was to stabilize farmer incomes, and that, in the long-term, such a scheme must be self-balancing. They also agreed to use some of the accumulated reserves as loans for dairy factory modernization.

The return to free trading in 1954 was met by a weak English market. Despite the fact that production costs had risen, the Commission responded to the market situation by reducing the guaranteed price. Hence, market realizations were emphasized by them as a price determining factor.

Dissatisfaction with the price-determining process led to a Committee of Enquiry being set up. As a result of this, an Amendment to the 1947 Act was passed in 1956. This Amendment removed the power of price determination from the Commission, and vested it in a new Dairy Products Prices Authority. When setting prices, the Authority was required to consider current price levels, ruling price levels for other farm products and the cost of production. The Commission was bound to purchase all production offered to it at the guaranteed price.

Overseas dairy prices fell sharply in the next two years, depleting the industry's accumulated funds, which led to a reappraisal of the guaranteed price procedure. The Dairy Board at this time maintained that the guaranteed price structure should be more closely related to market realizations. Since the Dairy Products Marketing Commission was now obliged to purchase all

production offered to it at guaranteed prices, it attempted to influence production patterns indirectly by developing an advisory service on market trends for manufacturers.

2.3.1.5 1961 to 1985

In 1961, the Dairy Products Marketing Commission, which was the then marketing authority, and the then Dairy Board, whose function for the previous twenty-five years had been that of developing industry unity, were amalgamated. The new authority, was the New Zealand Dairy Production and Marketing Board, which was renamed the New Zealand Dairy Board in 1966. The 1961 Act, subject to minor amendments, remains in force at the present time. The Dairy Products Prices Authority continues to set guaranteed prices, based on criteria set out in the Dairy Board Act 1961.

The major function of this new Board was to acquire and market dairy produce manufactured in New Zealand and intended for export. To this end, it was endowed with the powers necessary to achieve this.

The New Zealand Dairy Board, therefore, markets dairy products on behalf of the industry. Distribution methods vary, but in general, the Board tends to set up subsidiary companies in individual markets, which act as marketing agents and wholesalers. In certain countries, the Board will appoint local agents or negotiate directly with individual companies.

To facilitate these marketing arrangements, the Board co-ordinates necessary ancilliary services such as shipping. Its other activities include promotion, research, dissemination of information, financing dairy company development and joint ventures, and stock improvement. It also administered the Government's supplementary minimum prices scheme from 1978 to 1985.

The guaranteed price scheme in the dairy industry operates at the manufacturing level. Originally, a basic dairy factory purchase price was set for butter. Similarly, a basic purchase price was offered for cheese such that returns to butter and cheese producers were essentially equalized. As alternative milk products have increased in importance, these have been introduced into the guaranteed prices scheme. Despite the firmly entrenched principle of equalized payouts, differentials to guide production have been built into purchase prices on occasion.

In recent years, the Dairy Board, in common with agencies in other agricultural industries, has placed increasing emphasis on a marketing orientation to its activities. Market research has been given increased emphasis, and this has influenced promotion and product research. The Board, as part of its marketing plan, is also committed to a programme of market diversification to reduce its dependency on particular markets, and to increase its market flexibility.

The current market environment facing the dairy industry has been characterized by a build-up of

international stocks, and by restricted access to traditional markets. In 1981/82, the Dairy Board bought up stocks of U.S. butter in an attempt to manage the international market. More recently, there were industry and Board suggestions that new suppliers into the industry be restricted (New Zealand Dairy Board, 1985), and such a policy has been implemented by a moratorium on the acceptance of new supply.

2.3.2 Economic Policies of Statutory Institutions

The original Dairy Produce Control Board operated for a decade from 1924. Its activities, outlined in Section 2.3.1.2 above, indicate that the focus of this Board was on demand-shifting policies through product advertising, and supply-shifting policies through cost-reducing activities. Initially, a primary economic policy was also price-fixing and stabilization, through the acquisition of produce, thereby allowing for co-ordinated selling and the exercise of market power. However, these latter policies appeared to be unsuccessful, and the Board concentrated on the less interventionist demand and supply-shifting policies.

The economic policies undertaken from 1934 to 1945 by the statutory successor to the Dairy Produce Control Board, the Primary Products Marketing Department, were similar to those originally envisaged for the Dairy Produce Control Board. The Department continued the demand and supply-shifting policies of the original Board. However, in addition to these, it resurrected the

price-fixing and price stabilization policies. The price-fixing policy was facilitated by the compulsory acquisition of produce which was disposed of by the Department, and payouts to producers were then equalized. The price stabilization policy was put into operation by the guaranteed price scheme.

At the conclusion of the Second World War, the Dairy Products Marketing Commission, which operated until 1961, continued these economic policies of the Primary Products Marketing Department. However, it became obvious that the purpose of the guaranteed price scheme was unclear. Uncertainty existed as to whether it was a self-balancing price stabilization scheme, or whether an element of subsidy was incorporated into it. In 1956, this price-setting function was removed from the Commission. Since the Commission was now obliged to accept all produce offered to it at a guaranteed price, this placed greater pressure on it to consider policies which would obtain the maximum market realizations possible by exploiting demand opportunities on the export market.

In 1961, the New Zealand Dairy Board superseded the Commission, and this new organization continued the economic policies instituted by its predecessors. However, in addition to these demand- and supply- shifting policies, and their administration of the price-fixing, stabilizing and pooling policies of the Dairy Products Prices Authority, the new Board, which is still operating, adopted a more sophisticated approach to the marketing of dairy produce.

It has engaged in market diversification activities, and appears, as evidenced by its attempts to influence manufacturing trends, to be attempting to exploit demand opportunities more fully by market segmentation policies. In fact, Veeman (1972) has found that its allocation of produce to the butter and cheese segments of the United Kingdom market was not incompatible with the behaviour of a price-discriminating monopolist. In addition, promotion and product development are being geared to specific market segments.

Recently, the Board has attempted to influence global prices by supply management in the international market. There have also been industry suggestions that increased output is reducing producer returns, and new entrants to the industry have been restricted, but as yet, the restriction of supply by existing producers is not an economic policy practised by the Dairy Board.

The evolution of the above economic policies which have been practised by statutory institutions operating in the dairy industry since 1921 are summarized in Figure 2.3.

2.3.3 Objectives of Statutory Institutions

Given the changes in market environment prior to, during, and immediately after the First World War, which led to the establishment of the Dairy Produce Control Board, it seems plausible to conclude that the intended objectives of this new institution were to increase prices to producers, and to stabilize these prices.

	DAIRY PRODUCE CONTROL BOARD 1921-1934	PRIMARY PRODUCTS MARKETING DEPARTMENT 1934-1947	DAIRY PRODUCTS MARKETING COMMISSION 1947-1961	DAIRY BOARD 1961-1985
<u>OBJECTIVES</u>	INCREASE PRODUCER RETURNS STABILISE PRICES	INCREASE PRODUCER RETURNS STABILISE PRICES EQUITY AMONG PRODUCERS	STABILISE PRICES INCREASE PRODUCER RETURNS EQUITY AMONG PRODUCERS	INCREASE PRODUCER RETURNS EQUITY AMONG PRODUCERS STABILISE PRICES (DPPA)
<u>EFFECTIVE POWERS</u>	POWER TO UNDERTAKE RESEARCH & PROMOTION ON BEHALF OF INDUSTRY POWER TO CO-ORDINATE SHIPPING	POWER TO ACQUIRE AND CONTROL THE MARKETING OF ALL EXPORT PRODUCE POWER TO SET PRICE	POWER TO ACQUIRE AND CONTROL THE MARKETING OF ALL EXPORT PRODUCE POWER TO SET PRICE (DPMC TO 1956; DPPA AFTER)	POWER TO ACQUIRE AND CONTROL THE MARKETING OF ALL EXPORT PRODUCE POWER TO SET PRICE (DPPA)
<u>ECONOMIC POLICIES</u>	DEMAND-SHIFTING (PROMOTION) SUPPLY-SHIFTING (RESEARCH, CO-ORDINATED SHIPPING) DEMAND SEGMENTATION (GRADING - GOVERNMENT)	AS FOR DAIRY PRODUCE CONTROL BOARD PRICE FIXING (GUARANTEED PRICE) PRICE STABILISATION AND PRICE POOLING (GUARANTEED PRICE, CONTROL OF MKT'ING)	AS FOR PRIMARY PRODUCTS MARKETING DEPARTMENT (POLICIES INSTITUTED THROUGH GUARANTEED PRICES ADMINISTERED BY DPPA AFTER 1956)	AS FOR DAIRY PRODUCTS MARKETING COMMISSION MARKET DIVERSIFICATION INTERNATIONAL PRICE INFLUENCING MARKET SEGMENTATION

FIGURE 2.3: THE EVOLUTION OF STATUTORY MARKETING IN NEW ZEALAND'S DAIRY INDUSTRY

The actual economic policies adopted by the Board during its period of operation from 1924 to 1934 seem to be consistent with these objectives. However, they also tend to suggest that emphasis was placed on the broader objective of increasing producer returns as opposed to prices. For example, in the long-run, adopting cost-reducing policies may reduce price where demand is less than perfectly elastic, but increase returns if demand is not inelastic.

As with the Dairy Produce Control Board, it appears from the market environment and the economic policies followed by the Primary Products Marketing Department from 1934 to 1945, that its primary objective was to increase returns to its producers. However, the secondary objective of stabilizing producer prices appears to have been given higher priority than with the Dairy Produce Control Board. In addition, the equalization of returns to producers implies an acceptance of an objective of equity of returns, a concept which they had been unhappy to endorse a decade previously.

The above objectives of the Primary Products Marketing Department also appear to apply to the Dairy Products Marketing Commission which operated from the end of the Second World War to 1961. The dissension over the guaranteed price scheme noted previously appears to have arisen over uncertainty as to whether the primary purpose of this policy was to increase producer returns, through subsidy if necessary, or to stabilize prices. The price stabilization objective of the scheme appeared to be

dominant by 1961, by which time it was administered by the Dairy Products Prices Authority.

The New Zealand Dairy Board, which has operated since 1961, also appears to have adopted the objectives of its predecessors. However, during this current period, the objective of increasing returns seems to have been emphasized more than the objective of price stability, since the method of stabilizing prices was accepted by then. The principle of equity among producers appears to have been well-entrenched when the Dairy Board came into being, and it continues the policy of equalized factory payouts which distributes returns from overseas markets across all co-operatives, although it will override this objective to some extent when it conflicts with that of increasing producer returns.

The relative emphasis on these objectives by statutory agencies over time is shown in Figure 2.3.

2.3.4 Effective Powers of Statutory Institutions

The experience of the Commandeer during the First World War relative to the periods of free enterprise marketing before and after it, suggested to producers that absolute control of marketing by a statutory authority was desirable. Although the new Board which began operation in 1924 was endowed with wide-ranging powers to control the export marketing of dairy produce, its short-lived experiment with these powers suggests that its effective powers were somewhat more limited than its actual legislative powers. Therefore, during the late 1920's and

early 1930's, its effective powers were restricted to those necessary to carry out promotion and research activities on behalf of suppliers, and to those necessary to co-ordinate the movement of produce to market.

However, the onset of the Depression in the 1930's generated a crisis in the dairy industry which led to the abandonment of the basic free enterprise system operating until then. From 1934 to 1945 the Primary Products Marketing Department exercised the wide-ranging powers previously conferred on, but not utilized by, the Dairy Produce Control Board. It was responsible for the marketing of all export produce, and returned producers a price based on market realizations and a number of other factors. Therefore, its effective powers were such that it could control the entire marketing process, including the pricing and destination of produce.

The end of Government control of dairy products marketing in 1945 did not see a return to free enterprise marketing. Therefore, by 1947, the principle of controlled marketing of export produce appeared firmly entrenched in the dairy industry. Hence, the effective powers of the Dairy Products Marketing Commission were such that it controlled the entire marketing process. However, towards the end of the period, these powers were constrained to the extent that the Commission was forced to accept all product offered to it at guaranteed prices set by the Dairy Products Prices Authority.

No changes in the effective powers of the statutory marketing authority occurred when the powers of the Dairy

Products Marketing Commission were transferred to the New Zealand Dairy Board in 1961. That is, the current Board still effectively controls the entire marketing process in the export sector, with guaranteed prices being set by the Dairy Products Prices Authority.

The effective powers practised by each of these institutions is outlined in Figure 2.3.

2.3.5 Summary of Objectives, Effective Powers and Economic Policies of Statutory Institutions

From Figure 2.3, it is obvious that the long-term objectives of agencies operating in the dairy industry are to increase producer returns, to stabilize prices and to provide equity among producers. These objectives appear to be quite stable, although the relative emphasis on each of them may change over time, according to the prevailing market environment. The current emphasis appears to be on increasing producer returns.

Over the last fifty years, the effective powers of marketing agencies in the dairy industry have also remained relatively stable, although the price-determining function was removed from the marketing authority to an independent pricing authority (Figure 2.3). These agencies have been empowered to acquire and market all dairy produce, and have been responsible for ancillary services such as product handling, pooling, transport and storage, and for insurance, processing, packaging, promotion and research. These agencies have never had the legislative authority to control the level of industry

output, although it has been implied by industry leaders that such powers might be desirable.

Although, as noted above, the long-term objectives of agencies operating in the dairy industry have remained stable over time, economic policies through which such objectives may be achieved have altered as market conditions in the industry have changed. Initially, demand and supply-shifting activities and co-ordinating activities, which are policies involving minimum intervention, were considered adequate for increasing returns and stabilizing prices. However, with the onset of global depression, more direct methods of achieving these objectives were employed through the guaranteed price scheme. More recently, with the build-up of international stocks and restricted access to traditional markets, emphasis has been placed on exploiting demand opportunities more fully through market segmentation policies.

2.4 THE EVOLUTION OF STATUTORY STRUCTURES IN THE MEAT INDUSTRY

2.4.1 The Market Environment and Statutory Institutional Activities

2.4.1.1 Prior to 1921

Statutory involvement in the export sector of the New Zealand meat industry began with the Slaughtering and Inspection Act 1900, which provided for the establishment of abattoirs and the licensing of meat export

slaughterhouses. The primary purpose of the Act was to ensure that sanitary conditions of slaughter were being observed.

Until 1915, a free enterprise system of marketing operated in the meat industry. However, price falls in the United Kingdom market in 1909 led to industry attempts in New Zealand to form a regulatory marketing institution, but the proposals were defeated.

For a five year period from 1915 to 1920, all meat available for export from New Zealand was purchased by the United Kingdom Government under the Imperial Commandeer. This meant a secure protected market for producers, where all export meat was sold at fixed prices. Over this period, production increased, but shipping difficulties led to a build-up of stocks in New Zealand.

2.4.1.2 1921 to 1939

The return to free enterprise marketing in 1921 coincided with low prices on the United Kingdom market. These were a result, in part, of two seasons' New Zealand production arriving on the British market simultaneously. This price fall, along with rising production and shipping costs, led to renewed calls for statutory intervention in the market.

As a result of these pressures, the Meat Export Control Act 1921-1922 was passed. This led to the establishment of the New Zealand Meat Producers' Board, which appeared to be vested with power to control exports and to approve export slaughtering facilities. It was

further empowered to act as sole agent in negotiating shipping contracts, and to organize grading, pooling, storage, disposal, insurance and advertising of New Zealand export meat. Its operations were to be financed by a levy charged on all exported meat. This Act, subject to Amendments subsequent to 1922, remains in force at present.

Despite the fact that the newly-established Meat Board appeared to be authorized to export meat, it did not exercise such authority initially, and major marketing functions continued to be carried out by private firms, with the Board maintaining a watch-dog attitude towards the slaughter and export costs of meat. In addition, the Board established and supervised grading standards, negotiated freight reductions, regulated meat export flows, initiated overseas market surveys and statistical recording, and carried out advertising.

During the Depression, market prices for meat were low, and, in the latter half of the 1930's, quotas on meat imports into the United Kingdom were in force. This economic climate led to suggestions of greater intervention in the meat industry, first with the 1934 Agriculture (Emergency Powers) Act, and then with the 1936 Primary Products Marketing Act. However, this latter Act was not activated with respect to meat products initially.

In 1939, the Meat Act was passed, which superseded the 1900 Act, and, in addition to hygiene, introduced economic criteria for approving applications for the extension of export works. The successors to this 1939 Act, along with

the 1922 Meat Export Control Act and the successors to the 1955 Meat Export Prices Act, provide the current statutory authority to intervene in the meat industry.

2.4.1.3 1939 to 1954

With the outbreak of war, the 1936 Primary Products Marketing Act was invoked and the marketing of export meat, under the bulk purchase agreement with the United Kingdom, became the responsibility of the Primary Products Marketing Department. This reduced the Board's activities to the supervision of grading and maintaining a watch-dog role on slaughtering costs on behalf of producers. At the conclusion of the war, the bulk service contract was extended until 1954, and in 1948, the Board took over the administration of the agreement.

In the early years of the contract, price increases were negotiated with the United Kingdom Government. The Primary Products Marketing Department then paid a fixed f.o.b. contract price to exporters, who in turn paid producers the corresponding schedule price. However, in the early years, these were not fully returned to producers. Instead, as part of the Government's economic stabilization policy measures, increases in price were credited to the Meat Industry Stabilization Account (later renamed the Meat Industry Reserve Account). This account continued to accumulate reserves, thereby resulting in producer dissatisfaction, and from the early 1950's no further price increases were paid into it. It was then

used to make loans to fertilizer, meat processing and topdressing companies.

With the expiration of the bulk purchase contract, export meat trading reverted to its previous basis of independent selling, with the Board resuming its pre-war functions.

2.4.1.4 1954 to 1971

From the conclusion of the bulk purchase contract until the 1970's, export meat marketing remained in the hands of private enterprise, with the New Zealand Meat Producers' Board maintaining its watch-dog role of schedule monitoring and advising producers to sell on own account or through the pools if it felt the meat exporters' schedule was unrealistic.

Towards the end of the bulk purchase agreement, the Board began to press for minimum prices for export meat which would utilize the meat industry reserve funds. This led to the Meat Export Prices Act 1955, which resulted in prices being set pre-seasonally by a Meat Export Prices Committee. Criteria used when setting prices were previous prices, market prospects, other farm costs, and the cost of living index. The scheme was envisaged as a deficiency payments scheme, rather than as a price stabilization one.

At the beginning of this period, concern had begun to mount over the dependence on the United Kingdom market, and the lack of diversification towards other markets. This led to the setting-up of the Meat Export Development

Company (NZ) Ltd, or Devco as it is commonly known, in 1960. This Company was given, and still retains, the sole rights to market frozen sheepmeat to the North American market. Its existence was facilitated by the 1959 and 1962 Amendments to the 1922 Act, which clarified the Board's powers to control exports by limiting them to areas of market development. During the 1960's, the financial losses of Devco generated considerable controversy. However, it appears that the diversion of supply from the United Kingdom to the North American market resulted in a strengthening of U.K. prices (Edwards, 1970).

A further attempt to accelerate the diversification of exports was made with the 1966 Amendment to the Meat Export Control Act. As a result of this, a Market Development Committee was constituted, and this Committee set diversification targets for lamb exports. Diversification penalties and bonuses were introduced to encourage individual companies to attain targets. This scheme was eventually discontinued in 1980.

A further development during this period occurred with the passage of the Meat Act 1964, which consolidated the 1939 Act. Under this Act, export slaughter-houses were granted licences only on the condition that they adopt an open-door policy, whereby licencees were obliged to process all stock offered to them and, if necessary, to arrange for the marketing of the meat on the offerer's behalf.

The Board also supported research activities during this period. In conjunction with the Freezing Companies Association, it financed the establishment of the Meat Research Institute whose aim was to foster, promote and undertake research in the meat and wool industry.

2.4.1.5 1971 to 1982

The 1970's marked a period of selective intervention by the Board in the market. The first occasion was in 1971, when the Board responded to a low opening lamb schedule by entering the market to trade in lamb. It did so with the authority of a 1971 Amendment to the 1922 Act, which overrode previous amendments, and allowed the Board to sell sheepmeats in any market. This authority was extended in 1974 to cover beef and veal.

Since then, the Board has again intervened in the market on a number of occasions during market downturns. In 1974, it offered its own schedules for beef, ewe and lamb, which it was then able to increase and to guarantee for another season with the assistance of Government who established a Meat Income Stabilization Account with an initial grant. This account was to be administered by the Board. Late in 1975, the Board again intervened, this time in the mutton market. The next occasion was in 1977 in the London market, where the Board purchased carcasses for resale later in the season under the auspices of Meatmark Ltd. It also intervened in the mutton market in the next season.

During this period, the Board also consolidated its powers to control the conditions of export to specific destinations. A 1975 Amendment to the 1964 Meat Act give the Board power to impose conditions on meat exporters' licences, such as limitations on meat type and destination, with the aim of assisting orderly marketing. This Amendment was then used to restrict the number of exporters in the West German lamb market. The growth in importance of markets such as the USSR and the Middle East led to an increased role for the Board in contract negotiation. In 1980, it signed a contract with Iran, and then appointed one exporter to manage this contract.

Changes to the minimum prices scheme also occurred in the 1970's. In 1976, through an updated Meat Export Prices Act, this scheme was transformed from a pure deficiency payment to one with deficiency payment and stabilization aspects. The criteria under which minimum prices were set were widened, and, in addition, a trigger price was set, above which levies were imposed. These were to be used to offset supplements payable when the market price was below the minimum price. The industry reserves were no longer available for supplementation. A further dimension to the Board's stabilization activities was added in 1978, when it found itself required to administer the Government's supplementary minimum prices scheme.

Further developments in the freezing sector of the industry occurred in 1981, with the passage of another Meat Act, which delicensed the industry. This allowed

slaughtering facilities to be freely established, subject to open-door arrangements.

2.4.1.6 1982 to 1985

The sheepmeats industry faced a difficult year in 1982. As a result, the Board intervened in the mutton market and eventually bought up 90 percent of the season's kill. Later in the season, the lamb schedule fell, and various intervention measures were taken including the Board taking responsibility for high-risk product, and reactivating Meatmark Ltd.

As the crisis in the industry deepened, a Meat Industry Task Force was set up to enquire into the system of meat marketing. The Task Force stressed a greater emphasis on a marketing approach, and recommended that the Board act as the primary exporter of carcasses and primal cuts, and that private exporters be licensed to acquire carcasses and primal cuts for export and further processing in accordance with an overall marketing plan (Meat Industry Task Force, 1983). In addition, it recommended setting-up a Meat Industry Council to formulate an industry strategic plan and to formulate and monitor annual marketing plans with the Board.

In the meantime, the Board had purchased all sheepmeats that season at a schedule based on Supplementary Minimum Prices. This essentially reduced the exporters' role to that of commission agents for the Board. Market realizations in 1983 were so low that support levels for meat prices were a record.

The Board then implemented the proposals of the Meat Industry Task Force and refined earlier proposals to establish an export pools system of payment for sheepmeats under Board control. It initially assumed responsibility for the carcass and primal cut segment of the market, although after a short period, it then decided that lamb carcasses and primal cuts to be sold to specific regions would be handled by a group marketing structure (N.Z. Meat Producers' Board, 1985a). Under this scheme, five groups of exporters were formed with only one exporter from each group being allowed to operate in each market.

Also in line with the Task Force recommendations, the Board placed increasing emphasis on product development and promotion as components in its more market-led strategy. It continued its cost-reducing activities, with reviews of shipping costs and grading. This latter review resulted in a new grading system in 1983/84. The newly-established Meat Industry Council also showed interest in cost issues, since it commissioned a report on cost competitiveness in export meat processing (N.Z. Meat Producers' Board, 1985b).

Despite this experiment in Board control, it became obvious at the beginning of the 1985/86 season that the international sheepmeats market was very depressed. As a consequence, the Board announced correspondingly low prices to producers. However, at this time, the Board's trading operations were subject to criticism by the meat exporting companies, and pressure by the Government to return to private enterprise marketing. Therefore, in

late 1985, the Board announced its intention to hand the marketing of sheepmeats back to the private meat exporters. The current situation is highly uncertain, although it appears that rights to market to certain areas will be allocated to specific exporters or groups of exporters.

2.4.2 Economic Policies of Statutory Institutions

The initial activities of the New Zealand Meat Producers' Board from 1921 to 1939 shows that its activities were largely focussed on supply-shifting economic policies through cost-reducing activities and its watch-dog role on processing and marketing costs. Minor demand segmentation was instituted through grading. In addition, it sought to influence price by regulating the flow of meat onto the United Kingdom market.

However, the emphasis changed from 1939 to 1954, when the Primary Products Marketing Department and, in the latter phases, the New Zealand Meat Producers' Board, had statutory responsibility. During this period, their primary economic policy was price-fixing through their negotiating roles in the bulk purchase contract and subsequent setting of prices to exporters. In addition, an income stabilization policy was imposed on the meat industry by Government and administered by these organizations. Hence, the policies pursued by statutory agencies over this period were much broader than those followed previously. However, the Board still continued

its supply-shifting policy through pursuing its watch-dog role on slaughter costs.

During the period 1954 to 1971, the Board began to place greater emphasis on pricing policies. Its schedule monitoring was designed to influence export prices by precluding monopsonistic exploitation of producers by meat exporters. This policy was given a further statutory boost by the open door licensing policy, which safeguarded producer rights to export on own account. More directly, it attempted to boost overall returns to producers through its supply diversion strategies. Prices were further increased by the minimum prices scheme, which was essentially a deficiency payment. In addition, emphasis continued to be placed on supply-shifting cost-reducing policies by the Board, which now included its active support for research into the industry.

In the fourth period under consideration, 1971 to 1982, the Board continued its emphasis on pricing policies, albeit with stronger market instruments. Its selective intervention in the market, including its storage and resale activity, was designed to force up prices. Similarly, the restriction of competition between exporters in specific markets through licensing was expected to increase prices and reduce costs. Supply-shifting policies through cost-reduction in the industry were continued with delicensing in the freezing industry being for this reason. For the first time, a stabilization policy was enacted by the Board with the transformation of the minimum prices scheme from a pure

deficiency payment to a hybrid stabilization and deficiency payment scheme.

Pricing policies continued to dominate the economic behaviour of the Board after 1982 and before the return to private enterprise in late 1985. However, over this period, it was felt necessary for the Board to assume entire control of the sheepmeats marketing system in order to arrest the fall in price to producers. Although this action was partly for long-term price-stabilizing reasons, it should be viewed basically as a deficiency payment, since Government supported prices through its Supplementary Minimum Prices Scheme. In its marketing strategy, the Board placed increasing emphasis on market segmentation policies, through promotion and product development and the ability to control the volume of product allocated to specific destinations. Its promotion strategies can also be interpreted as having a demand-shifting element. The Board also continued its long-standing interest in supply-shifting policies through cost-reducing strategies.

The evolution of these economic policies is summarized in Figure 2.4.

2.4.3 Objectives of Statutory Institutions

The preamble to the Meat Export Control Act 1921-22 stresses that the reason for setting-up the New Zealand Meat Producers' Board was the depressed market environment facing producers. Falling net returns had occurred because of falling prices and rising costs. This suggests

	NEW ZEALAND MEAT PRODUCERS' BOARD (NZMPB) 1923-1939	PRIMARY PRODUCTS MARKETING DEPT 1939-1948 NZMPB 1948-1954	NZMPB 1954-1971	NZMPB 1971-1982	NZMPB 1982-1985
<u>OBJECTIVES</u>	INCREASE PRODUCER RETURNS	MAINTAIN AND LATER INCREASE PRODUCER RETURNS	INCREASE PRODUCER RETURNS	INCREASE PRODUCER RETURNS STABILISE PRICES	INCREASE PRODUCER RETURNS EQUITY AMONG PRODUCERS STABILISE PRICES
<u>EFFECTIVE POWERS</u>	POWER TO NEGOT. COST REDUCTIONS & ENFORCE GRADES POWER TO PROMOTE POWER TO REGULATE SHIP- MENTS POWER TO LEVY	AS FOR NZMPB 1923-1939 POWER TO SET PRICE OF EXPORTS	AS FOR NZMPB 1923-1939 LIMITED POWER TO CONTROL EXPORT VOLUME TO SOME DESTINATIONS	AS FOR NZMPB 1923-1939 INCREASED POWER TO CONTROL EXPTS TO SOME DESTIN'S POWER TO BUY & SELL	CONTROL OF ALL SHEEPMET MARKETING EITHER DIRECTLY OR BY DELEGATION
<u>ECONOMIC POLICIES</u>	SUPPLY-SHIFTING (WATCH-DOG ON COSTS) DEMAND-SHIFTING (PROMOTION) PRICE INCREASING (CO-ORDINATED SHIPPING) MINOR DEMAND SEGMENTATION (GRADING)	PRICE FIXING (BULK PURCHASE NEGOTIATION) SUPPLY-SHIFTING (REDUCING SLAUGHTER COSTS) MINOR DEMAND SEGMENTATION (GRADING)	PRICE INCREASING (SCHEDULE MONITORING, OPEN DOOR POLICY) SUPPLY DIVERSION PRICE SUPPORT (MINIMUM PRICES) SUPPLY-SHIFTING MINOR DEMAND SEGMENTATION (GRADING)	PRICE STABILISATION (INTERVENTION) PRICE INCREASING (INTERVENTION, EXPORT LIC'NG) MARKET DIVERS. SUPPLY-SHIFTING (DELICENSING FREEZING INDUS.) MINOR DEMAND SEGMENTATION (GRADING)	PRICE STABILISATION & INCREASING (CONTROL OF MARKETING) PRICE SUPPORT MARKET SEGMENT'N (PRODUCT DEVPT., GRADING, PROM.) DEMAND SHIFT (PROMOTION) SUPPLY SHIFTING (REDUCING COSTS)

FIGURE 2.4: THE EVOLUTION OF STATUTORY MARKETING IN NEW ZEALAND'S MEAT INDUSTRY

that the primary objective of the Board from 1921 to 1939 was to increase returns to producers. The demand- and supply-shifting and price influencing policies subsequently followed by the Board over this period were designed to increase prices and reduce costs, and therefore, support this objective.

During the war and the post-war reconstruction phase, the objectives of statutory institutions operating in the meat industry were influenced by Government's general economic objectives. Therefore, rather than aiming to increase producer returns, the objective of such organizations could be interpreted as attempting to maintain existing producer returns, while increasing industry returns which were to be redistributed to producers in the future. In the latter part of the period from 1939 to 1954, when transfers to the Meat Industry Reserve Account were discontinued, the primary objective once more appeared to be to increase existing producers' returns.

From 1954 onwards, the policies followed by the Board confirm that its paramount objective was to increase producer returns. During the period 1954 to 1971, producers also showed some acceptance of a principle of an equitable distribution of aggregate market returns among them, since the meat exporters' schedule would not reflect the differential returns from markets which would result from supply diversion strategies. However, this was qualified by the extent to which more direct marketing options were available to them.

This question of equity assumed increasing importance during the period 1971 to 1982, when operations in specific markets became more restricted, but the benefits from such a policy were distributed over those producers participating in the schedule system of pricing by exporters. During this period also, the objective of price stabilization had assumed some importance through the Board's intervention activities.

From 1982 to 1985, the emphasis on the principle of equity was further accentuated by Board control of marketing, since collective industry returns were distributed among individual producers. However, the objective of increasing producer returns remained the primary concern of the Board, while the stabilization objective, which had assumed some importance during the previous period, was de-emphasized.

This evolution in the relative emphasis on these objectives is shown in Figure 2.4.

2.4.4 Effective Powers of Statutory Institutions

Despite being endowed with powers to control the export marketing of meat, the New Zealand Meat Producers' Board chose not to activate these over the period 1921 to 1939. Therefore, its effective powers at this time were restricted to those necessary for the co-ordinating and bargaining activities which reduced costs and influenced prices, and to promotion activities, all of which were financed by a levy on meat exports.

Over the period of the bulk purchase contract from 1939 to 1954, the basic system of free enterprise marketing in the meat industry was abandoned, and replaced by statutory control of price and destination by the Primary Products Marketing Department initially, and later by the New Zealand Meat Producers' Board. Hence, exporters were effectively reduced to commission agents. Therefore, the Department and the Board assumed most of the powers which originally appeared to envisaged under the Meat Export Control Act 1921-22.

At the conclusion of the bulk purchase contract in 1954, the effective powers of the Board reverted to the minimum intervention pre-war level. However, over the period from 1954 to 1971, these effective powers increased as it gained authority to control, to some extent, the volume of exports to specific destinations.

This trend towards increasing control over export marketing initiated in the 1960's continued during the 1970's, with the intervention of the Board in the market on occasion, and with the increased use of licensing to control exports to specific destinations. With the assumption of powers to purchase the entire sheepmeat kill in 1982, and reallocating specific market segments to exporters under specific conditions, the Board effectively controlled the entire sheepmeats marketing process thereby allowing it to control the volume and destination of product as it thought fit. With the return of sheepmeats marketing to private enterprise, the current situation is very unclear, although it appears that the Board may

retain the privilege of allocating rights to certain market areas to specific companies, or that the companies may attempt to do this voluntarily.

The development of these effective powers over time is illustrated in Figure 2.4.

2.4.5 Summary of Objectives, Effective Powers and Economic Policies of Statutory Institutions

From Figure 2.4, it is obvious that the primary focus of the New Zealand Meat Producers' Board has always been, and still is, to increase returns to producers. In more recent years, stabilization of prices appears to have become a secondary objective. In addition, the recent period of Board intervention in the market suggests that the concept of an equitable distribution of market returns among producers had also become an accepted objective by that stage.

The effective powers of the Board have varied a great deal over the period of its operation. This is shown in Figure 2.4. Although it initially engaged in minor intervention activities, its powers increased during the period of the bulk purchase. Following the conclusion of that contract in 1954, its powers effectively reverted to the minimum levels prevailing previously. However, these effective powers have gradually increased in the last thirty years, as a result of activities such as the control of product flow to specific markets, selective intervention, and more recently, virtually complete control of sheepmeats marketing. Although the current

situation is unclear, it appears that the Board may, to some extent, retain control of the flow of product to some destinations through the allocation of rights to operate in these markets.

As noted with the dairy industry, the predominant long-term objective of the New Zealand Meat Producers' Board has remained stable over time, but the economic policies which it has adopted in order to achieve this objective has altered with changes in the market environment facing the industry.

The focus was initially on supply-shifting policies and minor demand segmentation through grading, although a price-fixing function was added to this during the bulk purchase contract. However, after the war, a shift in policies began to occur with price-increasing strategies through schedule monitoring, and price-support policies through the minimum prices scheme being adopted. In addition, attempts were being made to manipulate demand more fully with supply diversion.

These policies have continued since then, but more direct methods such as intervention, and more recently, acquisition, have been applied in an attempt to force up prices although this latter policy has now been abandoned. Recently, increasingly sophisticated techniques of market segmentation have been employed to further exploit demand opportunities.

2.5 THE EVOLUTION OF STATUTORY STRUCTURES IN THE WOOL INDUSTRY

2.5.1 The Market Environment and Statutory Institutional Activities

2.5.1.1 Prior to 1936

Before 1916, unregulated free enterprise typified the sale of New Zealand wool. From 1916 to 1920, the United Kingdom purchased the New Zealand clip under the Imperial Commandeer, which was administered by the New Zealand Wool Committee.

At the conclusion of the Commandeer, considerable stocks of Australian and New Zealand wool remained on hand. Therefore, the British Australia Wool Realisation Association Ltd (BAWRA) was formed in 1921 to dispose of these stocks. By 1924, this had been achieved at a profit to producers by controlling the flow of wool onto the market with price reserves on all lots. The New Zealand Wool Committee assisted in this orderly disposal by scheduling auction quantities of new season wool from 1921. BAWRA was wound up in 1924, and the industry returned to an essentially free-enterprise system until the onset of World War 2.

2.5.1.2 1936 to 1945

During the Depression however, international wool prices began to fall. This led to moves by the major wool-exporting countries and the United Kingdom to stimulate research into wool production and the promotion of wool use. In New Zealand, this led to the 1936 Wool

Industry Promotion Act which provided for the establishment of the New Zealand Wool Council. Similar bodies were set up in Australia and South Africa, and the three national organizations established and financed the International Wool Publicity and Research Secretariat. The New Zealand Wool Council's role was to administer levy funds, most of which were expended on the International Secretariat.

During the Second World War, an appraisal scheme, similar in principle to the Commandeer, was in operation. This was administered by the Primary Products Marketing Department. Each year, a weighted average price for New Zealand wool was negotiated, and returns for different wool types were calculated from this. In the latter war years, negotiated price increases were siphoned into stabilization accounts as part of government's general stabilization measures.

In 1944, a New Zealand Wool Board was established, which replaced the Wool Council. The Board took over the Council's functions of wool promotion and wool improvement.

2.5.1.3 1946 to 1952

By 1945, an international wool stockpile, equivalent to two season's production, had amassed. Following the example of 1921, an international 'Joint Organisation', with subsidiaries in producing countries, was set up. Its aim was to dispose of the U.K. stockpile and current

production in an orderly manner through reserve price operations.

The New Zealand subsidiary was known as the New Zealand Wool Disposals Commission, and began operation from 1946/47. The Commission's operating costs were covered by a levy on producers. However, over the period of the stockpile, prices rose and levy receipts exceeded operating costs, with the result that, by 1951, when the Joint Organisation was wound up, the Commission had accumulated substantial reserves.

During this period, the Wool Board continued to levy producers to support international wool promotion.

2.5.1.4 1952 to 1972

With the winding-up of the Joint Organization, much discussion took place on the possibility of setting-up and administering an international reserve price scheme for the wool industry. However, international support was not forthcoming, so New Zealand decided to develop a minimum prices scheme to stabilize New Zealand wool prices alone, utilizing the profits of the New Zealand Wool Disposals Commission.

The minimum price scheme was put into operation through the Wool Commission Act 1951. This Act facilitated the establishment of the New Zealand Wool Commission, which inherited the reserves of the New Zealand Wool Disposals Commission and New Zealand's share of the Joint Organization profits.

The Commission operated by setting minimum prices for each grade of wool at the start of the season. When necessary, it would bid its minimum price on individual auction lots, and stockpile any wool acquired in this manner. The stockpile would then be released onto the market during buoyant trading conditions.

Until 1966, market prices were generally above minimum price levels. In a few seasons where this was not the case, only small quantities of wool were bought, and these were quickly cleared. However, in 1966/67, wool prices fell dramatically, and the Commission purchased over one-third of the wool offered at auction. Prices were still low in the next season, with the result that the Commission adopted a two-tier scheme whereby it entered the auction at a lower buying-in minimum price, but supplemented growers' receipts to a higher price level.

The stockpile was gradually disposed of between 1968 and 1973. Until 1972, much of it was sold at a loss, but a rapid price rise in 1973 allowed a small gross profit to be made on overall sales. However, this was more than offset by stockpile expenses, and, in addition, further losses resulted from supplementation.

While the major function of the New Zealand Wool Commission over this period was to operate the minimum prices scheme, minor functions included participating in the Wool Auction Sales Committee, assembling and disseminating statistical information, and collecting grower levies which financed Wool Board operations. The

Wool Board was responsible for research and promotion functions.

2.5.1.5 1972 to 1977

In the late 1960's, concern was expressed over price fluctuations, cost inefficiencies and falling market share in the wool industry, leading to speculation on the appropriateness of the current wool marketing structure. This culminated in two Reports (Wool Marketing Study Group, 1967; Battelle, 1971). The first of these suggested that the New Zealand Wool Commission should have power to acquire all wool at guaranteed prices, which would then be sold through traditional channels. The later Battelle Report recommended direct selling by an aggressive marketing corporation. It supported the phasing out of the auction system through competition with existing channels rather than by acquisition.

In response to the recommendations in these Reports, the New Zealand Wool Marketing Corporation Establishment Company was set up which further recommended that the pending Corporation ultimately assume power to acquire and market the entire clip. However, grower support for an organization with such wide-ranging powers was not manifest, and the Wool Marketing Corporation Act of 1972 allowed acquisition only where a referendum of growers showed a majority of them to favour such a move. However, support for acquisition strengthened although no referendum was held, and the then Labour Government amended this provision in the Act in 1974, allowing the

New Zealand Wool Marketing Corporation to acquire the national clip without a referendum of growers, although grower support still needed to be manifest. For a discussion of this 'acquisition debate', see Chudleigh (1978).

The Wool Marketing Corporation continued the minimum prices scheme of its predecessor. In 1974, it introduced a policy of market intervention to replace its two-tier price scheme. This meant that its bidding level could now be above or below the grower minimum levels. In addition, more finely tuned intervention, known as strata-price control, could be used to prevent a drastic price decline in any particular sale.

For 18 months from 1975, price supplements were funded by Government grant. In mid-1976, however, they were funded by a levy on all wool sold above minimum price levels, following the introduction of the Wool Income Stabilization Regulations. In addition, all wool sold in excess of a specific trigger price attracted a 50 percent levy on the excess.

In 1976, the Corporation also introduced the 'Extra-Choice' scheme, whereby it would buy certain classes of wool at current market prices less the estimated cost of resale, and feed these purchases into the auction system at a later date. During this period, the Corporation also encouraged efficient marketing by subsidising new sample selling methods. In the latter part of its existence, it provided credit and wool storage assistance to New Zealand mills.

The Corporation's counterpart organization, the New Zealand Wool Board, continued to concentrate on promotion, research and development of wool and wool products, and efficiencies in production and distribution.

2.5.1.6 1977 to 1985

The Wool Industry Act of 1977 amalgamated the New Zealand Wool Marketing Corporation and the New Zealand Wool Board into a reconstituted New Zealand Wool Board. This legislation currently remains in force.

Stabilizing wool prices through its minimum prices scheme remains the predominant activity of the present Wool Board. Minimum prices are set annually, with regard being paid to market conditions, levels of reserves, the maintenance of production, and the previous minimum price. The Board may achieve minimum prices either by supplementing returns, or by intervening directly in the market. Although basic intervention price levels may be altered, this tends not to be done very frequently. Retention levies continue to be applied when the adjusted weighted average sale price exceeds the trigger price. In addition to its own pricing scheme, the Board administered the Government's supplementary minimum prices scheme while this was in operation.

Since 1977, the Board has also engaged in cost-reducing activities such as fostering sale by sample and by description, and negotiating freight rate reductions.

The IWS continued its international promotion activities on behalf of member producers. However, its

promotion activities are becoming less general, with increasing emphasis being placed on market research to identify demand trends and market niches. Co-ordinated promotion and product development strategies are then pursued in accordance with these market requirements.

2.5.2 Economic Policies of Statutory Institutions

From 1921 to 1924, the objective of BAWRA's orderly disposal of accumulated stocks was to prevent the drastic price decline which would have occurred if all wool had been released onto the market in 1921. Therefore, BAWRA could be said to have been pursuing an economic policy of medium-term price stabilization. During this period, the New Zealand Wool Committee sought to do the same on a shorter-term basis through its auction scheduling procedures. During the remainder of this period until 1936, the industry reverted to a free enterprise marketing system.

However, during the next period considered, 1936 to 1945, further statutory bodies arose. As an initial response to the low wool prices prevailing in the Depression, the New Zealand Wool Council, and its successor, the New Zealand Wool Board, assisted in an aggregate demand-shifting policy through the promotion activities of an international cartel. In addition, during the period of the Second World War, the Primary Products Marketing Department was engaged in an economic policy of price-fixing through the annual negotiated bareme weighted average price.

From 1946 to 1952, the major economic policy of the then statutory body, the New Zealand Wool Disposals Commission, like that of BAWRA twenty-five years previously, was medium-term price stabilization through the international price reserve buffer stock scheme which assisted in the orderly disposal of stocks. The Wool Board, on the other hand, continued to assist in a demand-shifting policy through international generic promotion.

During the period 1952 to 1972, the major economic policy of the Commission was to support prices through the minimum prices scheme, which was operated by quasi-buffer stock activities. It was not a true buffer stock scheme to the extent that it only had a floor associated with it. Meanwhile, the New Zealand Wool Board continued to participate in a demand-shifting policy through its association with the International Wool Secretariat.

In 1972, the New Zealand Wool Marketing Corporation was established. Initially, the predominant economic policy of the new Corporation was to continue the price support scheme of its predecessor. However, over the period 1972 to 1977, the scheme was transformed to a price stabilization measure. The Corporation also aimed to promote efficiency in the marketing system through the Extra-Choice scheme, thereby reducing peak pressures on the auction system. In addition, it was engaged in supply-shifting policies through supporting cost-reducing activities such as sample selling. During the period, the Wool Board continued its demand-shifting policies through

promotion by the IWS, and also increased its interest in supply-shifting policies through concentrating on research and development, and production and distribution efficiencies.

The New Zealand Wool Board and the New Zealand Wool Marketing Corporation were amalgamated in 1977. The new New Zealand Wool Board's major economic policy from 1977 to 1985 continued to be price stabilization, through a combination of both buffer stock and buffer price activities. In addition, it continued to emphasize supply-shifting strategies like the Corporation before it, by concentrating on sale by sample and by description, and by negotiating freight reductions. The continuing promotion activities of the IWS were designed to shift demand. However, over the period, the emphasis seems to have changed from a pure generic shift in demand to more sophisticated techniques for segmenting the market.

The evolution of these economic policies is summarized in Figure 2.5.

2.5.3 Objectives of Statutory Institutions

Prior to 1936, the primary objective of statutory bodies operating in the wool industry appeared to be the medium-term stabilization of prices. However, from 1936 to 1945, the objective of the demand-shifting policy of the New Zealand Wool Council was to increase producer prices and returns. Likewise, the objective of the Primary Products Marketing Department was also to increase, and later to maintain, producer prices through

	NEW ZEALAND WOOL COMMITTEE (1916-1936) BAWRA (1921-1924)	NEW ZEALAND WOOL COUNCIL (1936-1945) PRIMARY PRODUCTS MARKETING DEPT (1939-1945)	NEW ZEALAND WOOL DISPOSALS COMMISSION/ NEW ZEALAND WOOL BOARD (1946-1952)	NEW ZEALAND WOOL COMMISSION/ NZWB (1952-1972)	NEW ZEALAND WOOL MARKETING CORP./ NZWB (1972-1977) NZWB (1977-1985)
<u>OBJECTIVES</u>	PRICE STABILIS- ATION (MEDIUM TERM)	INCREASE OR MAINTAIN PRICES	PRICE STABILISATION INCREASE RETURNS	INCREASE PRODUCER RETURNS	STABILISE PRICES INCREASE PRODUCER RETURNS
<u>EFFECTIVE POWERS</u>	POWER TO ACQUIRE CLIP AND TRADE	POWER TO SET EXPORT PRICES POWER TO LEVY POWER TO PROMOTE	POWER TO TRADE POWER TO LEVY POWER TO PROMOTE	POWER TO TRADE POWER TO LEVY POWER TO PROMOTE AND RESEARCH	POWER TO TRADE POWER TO LEVY POWER TO PROMOTE AND RESEARCH POWER TO NEGOTIATE COST REDUCTIONS
<u>ECONOMIC POLICIES</u>	PRICE STABILISATION (RESERVE PRICE SCHEME, AUCTION SCHEDULING)	PRICE FIXING (NEGOTIATE BAREME AVERAGE) DEMAND-SHIFTING (PROMOTION)	PRICE STABILISATION (RESERVE PRICE SCHEME) DEMAND-SHIFTING (PROMOTION)	PRICE SUPPORT (MIN. PRICES SCHEME) SUPPLY-SHIFTING (COST REDUCING ACTIVITIES) DEMAND-SHIFTING (PROMOTION)	PRICE STABILISATION (BUFFER SCHEME) SUPPLY-SHIFTING (COST REDUCING) ACTIVITIES) DEMAND SHIFT (PROMOTION) MARKET SEGMENT'N (PROMOTION, PROD DEVELOPMENT)

FIGURE 2.5: THE EVOLUTION OF STATUTORY MARKETING IN NEW ZEALAND'S WOOL INDUSTRY

its price-fixing behaviour. The emphasis slightly changed again from 1946 to 1952, when the primary objective of the Zealand Wool Disposals Commission, like BAWRA twenty-five years previously, was medium-term price stabilization. The fundamental objective of the Board over this period appeared to be to increase returns.

The primary objective of the minimum prices scheme operated by the New Zealand Wool Commission from 1952 to 1972 appears to have been to increase producer returns, although a secondary objective of such a scheme could be considered to be price stabilization. The objective of the demand-shifting promotion activities of the Board through the IWS continued to be to increase producer returns.

From 1972 to 1977, the objective of the minimum prices scheme, now administered by the New Zealand Wool Marketing Corporation, seemed to change. Instead of emphasizing an increase in producer returns, its objective could more accurately be seen as stabilizing producer prices. As was the case in previous periods, the other policies of both the Corporation and the Board continued to have the objective of increasing producer returns.

As with the Corporation in its latter years, the major objective of the newly constituted New Zealand Wool Board from 1977 to 1985, seems to be to stabilize price through its price stabilization activities. The supply-shifting and demand-shifting policies adopted and supported by the Board were designed, as had been the case with its predecessors, to increase producer returns.

The development in these objectives is noted in Figure 2.5.

2.5.4 Effective Powers of Statutory Institutions

Over the period 1921 to 1924, BAWRA effectively acquired the Australian and New Zealand wool clip and stocks from the Commandeer period. Hence, through its price reserve system, it controlled the volume of supply flowing onto the international market. The industry then reverted to a private enterprise system until 1936.

From 1936 to 1945, statutory powers in the wool industry were rather mixed. The powers of the New Zealand Wool Council and the predecessor of the International Wool Secretariat were those necessary to levy producers and conduct global generic promotion. During the period of the Second World War, the powers of the Primary Products Marketing Department were much more considerable, with Government power to set prices returned to producers, and to control the flow of wool exports to the United Kingdom.

From 1946 to 1952, the Joint Organization acquired the total New Zealand clip, some of which it then sold to the trade. However, in later years, when the Commission bought stocks via reserve price operations, effective statutory powers were reduced to trading to facilitate the international buffer price scheme.

This effective power to selectively intervene in the market continued during the period 1952 to 1972 when the New Zealand Wool Commission was in operation. Over the entire period from 1946 to 1972, the New Zealand Wool

Board retained its power to levy to support international promotion.

Although the 1972 New Zealand Wool Marketing Corporation had more substantial legislative powers to control wool marketing than did its predecessor, its effective powers from 1972 to 1977 differed very little from those of the Commission, with power to selectively intervene in the market to support its stabilization activities. In addition, both the Corporation and the Board had the necessary power to levy and conduct minimum intervention demand- and supply-shifting activities.

With the amalgamation of the Board and the Corporation in 1977, the substantial legislative powers, including the power to control the sale, disposal and export of wool without a grower referendum, remained unchanged. However, it has not fully utilized such authority and its effective powers from 1977 to 1985 have remained similar to those of the preceding period.

The evolution of these effective powers is summarized in Figure 2.5.

2.5.5 Summary of Objectives, Effective Powers and Economic Policies of Statutory Institutions

The objectives over time of statutory institutions operating in the New Zealand wool industry are summarized in Figure 2.5. Over the years, the major objectives of such agencies have been to stabilize producer prices and to increase producer returns. However, the relative emphasis on each of these has varied according to the

market environment at the time, and currently, price stabilization appears to be the predominant objective of the two.

Figure 2.5 also illustrates the effective powers of statutory institutions operating in the wool industry. Although those operating before 1924 acquired the New Zealand wool clip, institutions operating in the industry since then have had much more limited effective powers. Their intervention in the market has been at low levels, with selective trading to support minimum prices or stabilization schemes, and levy activities to finance promotion and cost-reducing policies. Therefore, despite the fact that the New Zealand Wool Board currently has the power to acquire and dispose of the entire New Zealand wool clip, it is unlikely that it would do so.

Over the last sixty-five years, the wool industry has been characterized by stable long-term objectives and low levels of effective intervention. In common with other industries studied, the economic policies adopted by the various agencies in order to achieve these objectives have varied over time, but not to the same extent as that noted in the dairy and meat industries.

In general, price stabilization policies in one form or another have dominated agency activities. On occasion however, in times of national emergency or depressed market conditions, price-fixing and price support activities have taken precedence. A demand-shifting policy through promotion by an international cartel of producers also has a long history.

In recent years, increasing emphasis has been placed on supply-shifting policies which reduce costs. These include fostering new technologies such as sale by sample and by description, and negotiating freight reductions. In addition, promotion efforts have recently become more sophisticated by exploiting identified market segments and developing specific products appropriate to those segments, although no attempts appear to have been made to manipulate the level of output allocated to these market segments.

2.6 THE EVOLUTION OF STATUTORY STRUCTURES IN THE PIPFRUIT INDUSTRY

2.6.1 The Market Environment and Statutory Institutional Activities

2.6.1.1 1924 to 1939

The first hint of grower organization in the New Zealand pipfruit industry occurred in 1916 with the formation of the New Zealand Fruitgrowers' Federation. The aim of this body was to assist the development of the industry, and it was financed by a levy on producers.

In 1924, the Fruit Export Control Board came into existence. From 1926 to 1939, this Board was the sole legal exporter of pipfruit, and it handled growers' fruit on a 'trustee' basis.

Growers received guaranteed payments for export fruit, although the mechanics of this guarantee varied over the period (New Zealand Fruit Export Control Board, 1928,

1931, 1933, 1935), and during its latter stages was not sufficient to cover production costs. (New Zealand Fruit Export Control Board, 1938). In addition, the Fruit Export Control Board also exercised economies in preparation, transportation and marketing.

An attempt at stabilization was made in the late 1920's, when it was suggested by Government that growers contribute towards a guarantee reserve fund which would initially be subsidized by Government, which eventually hoped to cease its guarantee. However, this scheme only remained in operation until the change of Government, when it was decided that the reserves accumulated should be distributed.

During the immediate pre-war years, the industry was beset by problems of rising costs, inadequate returns to growers and a shortage of shipping space for exports. Consequently, the government offered the industry a completely guaranteed price, provided the industry agreed to statutory control over the marketing of all pipfruit. There was also the suggestion that the government would withdraw the existing export guarantee if this offer was not accepted. Therefore, the industry requested the Government to take marketing control and to pay the industry a guaranteed price sufficient to cover production costs and to give the producer a fair standard of living (New Zealand Fruit Export Control Board, 1938).

2.6.1.2 1939 to 1948

With the outbreak of war, export shipping space was further restricted to meat, wool and dairy products. The Internal Marketing Division of the Primary Products Marketing Department purchased the entire pipfruit crop and assumed responsibility for marketing.

It attempted to dispose of the fifty percent of the crop, which had hitherto been exported, onto the domestic market. It did this by lowering retail prices and instituting grading, cold storage and a national distribution system.

Since Government now controlled the marketing of pipfruit, the New Zealand Fruit Export Control Board was reduced to advisory status only.

2.6.1.3 1948 to 1985

After the war, the New Zealand Fruitgrowers' Federation sought an arrangement which would ensure orderly marketing in both local and export markets. As a result, the Apple and Pear Marketing Act 1948 was passed, and the New Zealand Apple and Pear Marketing Board came into operation in the same year. This Act was subsequently amended and consolidated in the Apple and Pear Marketing Act 1971. It has since been amended further.

The Board is obliged to purchase all fresh apples and pears offered to it at set prices, subject to minimum grade standards and certain harvesting and packaging procedures. It is then required to market that portion of

the crop which it acquires on either the domestic or the export market. This accounts for the majority of production, since gate sales, despite being a cause of some concern to the Board in the past, do not account for a high proportion of the crop. The Board is also responsible for acquiring and marketing imported apples and pears.

Before 1967, Government had set the guaranteed price on a cost of production basis. However, in that year, the Apple and Pear Prices Authority, established under the 1948 Act, was charged with determining an average guaranteed price, taking into account market prices, past guaranteed prices, costs of production and marketing, the stability of the industry, industry reserves, and the capital needs of the Board. In 1979, these criteria were adjusted in order to remove the ten percent restriction on the movement of the guaranteed price. While the Authority set the average guaranteed price, it was the responsibility of the Board to determine grower prices for individual grades, varieties and districts, which were consistent with the average price determinations of the Authority.

From 1977 to 1980, industry reserves built up substantially, and in 1981, an Amendment to the 1971 Act introduced a stabilization element into Board activities. In addition to the guaranteed price set each year by the Authority, the Board announced a supplementary price, which is an average of last season's market return and the return estimated for the coming season. However, when the

stabilization funds are low, a price lower than this average may be announced. If the market price is less than the guaranteed price, growers receive the guaranteed price. If it lies between the guaranteed and the supplementary price, then growers receive the market return, adjusted for a capital charge to finance Board operations. Finally, if the market price is greater than the supplementary price, growers receive the supplementary price plus fifty percent of the difference between the two, with the balance being used to build-up reserves to finance years of inadequate market returns.

With its longstanding ability to control the destination and end use of product, the Board has been in a position to exploit demand opportunities on behalf of producers, and there is evidence to suggest that its diversion of supply from the fresh to the process market has resulted in increased returns to producers (Rae, 1978).

In recent years, this exploitation of existing and potential demand outlets has become more sophisticated. During this period, the market environment has been characterized by increasing production from both New Zealand and its competitors, and by protectionist attitudes in major markets. The Board has responded to this situation by supplying a wide range of product to an increased number of markets, with emphasis on a high quality product. This demand-expanding strategy has been appropriately supported by promotion.

The Board has also supported the introduction of cost-reducing technology such as palletization in shipping and co-ordinating freight schedules. In addition, it has expanded cool-store and packing facilities to cope with increased production, and is fostering efficiencies in production.

2.6.2 Economic Policies of Statutory Institutions

An important economic policy of the Fruit Export Control Board from 1924 to 1939 was price-fixing. In the latter years of its existence, an attempt at price stabilization was made. However, this stabilization scheme was abandoned, and from 1937, a price-fixing policy based on a guaranteed price was in operation. During this period, the Board was also concerned with supply-shifting policies, through cost-reducing activities in the transporting and marketing of pipfruit.

From 1939 to 1948, the Primary Products Marketing Department essentially continued the pre-war economic policies of the Fruit Export Control Board. That is, it fixed prices to producers, and continued with a supply-shifting policy through cost-reducing activities.

When power was transferred from the Department to the Apple and Pear Marketing Board in 1948, this new statutory body continued its predecessor's price-fixing policy, through the guaranteed price scheme, where the fixed price was based on a number of factors. In recent years, however, this has been transformed to a price

stabilization policy with minimum and trigger prices. Over the period 1948 to 1985, the Board has continued the tradition of supply-shifting policies through cost-reducing techniques in production, transportation and processing.

In recent years, it has also adopted demand manipulating policies such as supply diversion and market diversification. These policies, and its emphasis on exploiting a wide range of market niches suggests that market segmentation policies are being practised by the Board. In addition, demand-shifting policies through promotion have been undertaken in order to expand aggregate demand.

Figure 2.6 summarizes this evolution of economic policies by statutory authorities in the pipfruit industry.

2.6.3 Objectives of Statutory Institutions

The price-fixing and supply-shifting activities of the Fruit Export Control Board which operated from 1924 to 1939 suggest that its primary objective was to increase producer returns. The attempt at price stabilization in the early 1930's indicates that stable prices were seen as a desirable objective, but this only appears to have been of minor importance, since the scheme was soon abandoned.

The Primary Products Marketing Department controlled pipfruit marketing from 1939 to 1948. Its activities suggest that its primary objective was to maximize returns to growers subject to the wartime constraints under which

	FRUIT EXPORT CONTROL BOARD (1924-1939)	PRIMARY PRODUCTS MARKETING DEPARTMENT (1939-1948)	APPLE AND PEAR MARKETING BOARD (1948-1985)
<u>OBJECTIVES</u>	INCREASE PRODUCER RETURNS STABILISE PRICES (A MINOR OBJECTIVE)	MAXIMISE PRODUCER RETURNS SUBJECT TO WARTIME CONSTRAINTS EQUITY AMONG PRODUCERS	INCREASE PRODUCER RETURNS STABILISE PRICES (A RECENT OBJECTIVE) EQUITY AMONG PRODUCERS
<u>EFFECTIVE POWERS</u>	POWER TO CONTROL MARKETING OF ALL EXPORT PRODUCE POWER TO SET PRICE	POWER TO ACQUIRE AND CONTROL MARKETING OF ALL EXPORT PRODUCE POWER TO SET PRICE	POWER TO ACQUIRE AND CONTROL MARKETING OF ALL EXPORT PRODUCE POWER TO SET PRICE (APPA)
<u>ECONOMIC POLICIES</u>	PRICE-FIXING (GUARANTEED PRICE) SUPPLY-SHIFTING (COST REDUCTIONS IN TRANSPORT AND MARKETING) PRICE STABILISATION (EARLY 1930'S)	PRICE-FIXING (GUARANTEED PRICE) SUPPLY-SHIFTING	PRICE-FIXING BECOMING PRICE STABILISATION SUPPLY-SHIFTING MARKET DIVERSIFICATION MARKET SEGMENTATION DEMAND SHIFTING (PROMOTION)

FIGURE 2.6: THE EVOLUTION OF STATUTORY MARKETING IN NEW ZEALAND'S PIPFRUIT INDUSTRY

the industry was operating. The principle of an equitable distribution of returns among growers appears to have gained industry support by that time.

From 1948, the New Zealand Apple and Pear Marketing Board took responsibility for marketing the pipfruit crop and has continued to do so since then. Its supply and demand-shifting policies, and market diversification and segmentation suggest that increasing producer returns continues to remain a primary objective.

In addition, the conversion of the guaranteed price scheme to a stabilization scheme suggests that price stabilization has now become an objective of the pipfruit industry. Once again, the principle of equity in returns among producers was accepted through the pricing system.

These institutional objectives are shown in Figure 2.6.

2.6.4 Effective Powers of Statutory Institutions

From 1924 to 1939, the Fruit Export Control Board, as the sole exporter of pipfruit, effectively controlled the entire marketing process, including the destination of produce. These powers continued when the Primary Products Marketing Department assumed control of the acquisition and marketing of the pipfruit crop from 1939 to 1948. This situation remained unchanged when the New Zealand Apple and Pear Marketing Board took over in 1948.

The Board, which has remained in operation for thirty-seven years, has effective control of all transportation, processing and marketing activities.

Another statutory agency, the Apple and Pear Prices Authority, has had the power to set prices since 1967.

The development in these effective powers is traced out in Figure 2.6.

2.6.5 Summary of Objectives, Effective Powers and Economic Policies of Statutory Institutions

Figure 2.6 summarizes the perceived objectives over time of statutory agencies operating in the pipfruit industry. This indicates that the long-term objectives of such institutions have been to increase producer returns and to ensure equity among producers in returns. In addition, the objective of stabilizing prices has recently achieved some importance. The first two objectives have remained stable over time, while the stabilization objective was emphasized in the early 1930's, then de-emphasized, and more recently, has resurfaced again.

During the last sixty years, the effective powers of marketing agencies operating in the pipfruit industry have remained extremely stable. Agencies have been empowered to acquire and market all pipfruit offered to them, and have been responsible for associated market activities such as transportation, packing, storage, processing, promotion and research. As with other industries studied, these agencies have not had the authority to control the level of industry output.

Until recently, the long-term economic policies practised by statutory agencies operating in the pip-fruit industry have remained quite stable. These agencies have

concentrated on price-fixing through the guaranteed price scheme, and supply-shifting policies through cost reductions in the marketing channel. In the last few years, however, the current Board has extended its activities to include a price stabilization policy and a demand-manipulating policy. The aim of the latter policy is to shift demand, diversify the market and segment it.

2.7 THE EVOLUTION OF STATUTORY STRUCTURES IN THE KIWIFRUIT INDUSTRY

2.7.1 The Market Environment and Statutory Institutional Activities

2.7.1.1 Prior to 1977

The expansion of the kiwifruit industry in the Te Puke district in the 1960's, led to fears that current markets could not take projected kiwifruit production. Therefore, in 1970, exporters and growers met and adopted a charter which set out the principles of a voluntary system of levies to finance kiwifruit promotion. Growers agreed to pay a levy of six cents per tray and exporters four cents per tray. The Kiwifruit Export Promotion Committee, with grower and exporter representatives, was set up to decide how levy funds should be used. The levy was voluntary, in that it was not backed by legislation, so growers agreed not to supply any exporter who did not pay the levy, and exporters agreed not to take fruit from a grower in the same circumstances.

Initially, promotion efforts were amateurish, but as sales, and therefore levies increased, efforts became more professional. The promoters carved out a niche for a new product with an image which was not too upper class to appear in supermarkets, yet up-market enough to carry a luxury price tag.

By the early 1970's, producers were receiving good prices for their product in the developing overseas markets, and this was attracting both new exporters and producers. Existing exporters became dissatisfied, since new entrants were receiving the benefits of promotion to which they had made no contribution. In addition, they felt that these newer exporters were buying and putting onto the market low quality fruit which established exporters would not handle. In addition, exporters appeared to be undercutting each other in overseas markets.

Growers were also concerned with the problem of potential oversupply, and could perceive a situation where, with falling prices, exporters would exit from the industry into new ventures, while growers would be locked into the industry with their considerable investment in vines.

While these developments were occurring, the Promotion Committee had begun to assume responsibility for matters beyond the scope of promotion. In 1973, it put forward a proposal to growers, which was accepted, that only the Hayward variety would be accepted for export. It also drafted recommendations on grading, packaging methods,

minimum fruit size and weight, and storage standards. In addition, it tried to forecast export volumes, but was unsuccessful in this, since growers often forward sold their crop to several exporters.

Industry demand for a marketing board began to emerge. It was envisaged that such an institution would be represented by growers and exporters, with sole authority to accredit exporters of kiwifruit, but no power to acquire fruit. A Steering Committee was set up and first met in June 1972, with the aim of converting the principles embodied in the Board proposal into a reality.

However, the then Labour Government was not enthusiastic about the proposal. In addition, exporters, who by that time had formed into the Kiwifruit Exporters' Association, withdrew their support for the concept, fearing the imposition of a single-seller marketing system on the industry. Some growers, who linked poor returns in the pipfruit and citrus industries to the activities of the Apple and Pear Marketing Board and the Citrus Marketing Authority, shared these views. As a consequence, the Steering Committee withdrew its support for the concept of exporter representation on the proposed Board.

By 1976, a schism had emerged in the industry. On the one hand, most growers were seeking total producer control, while on the other, exporters supported by a group of growers, were demanding no interference. In late 1976, the Minister of Agriculture made it clear to the Steering Committee that it had lost substantial grower

support and must, therefore, compromise. At the same time, he warned exporters that they would risk Government intervention if they continued to hold out against the establishment of some form of control authority. As a result, both parties agreed to the concept of a licensing authority with exporter representation. The Steering Committee was duly disbanded when the regulations were promulgated.

2.7.1.2 1977 to 1985

In 1977, the Kiwifruit Marketing Licensing Regulations came into force, thereby establishing the New Zealand Kiwifruit Marketing Licensing Authority, renamed the New Zealand Kiwifruit Authority in 1981. Authority for the Regulations emanated from the Primary Products Marketing Act 1953.

As envisaged, the predominant role of the Authority is the licensing of exporters, with the objective of promoting orderly marketing such that quality is maintained and exporters do not undercut each other in their markets. In 1974, fourteen exporters were in the business, as well as some growers who were exporting on their own account. In the first year of operation of the Authority, nine export licences were granted, but by 1982, this number had been reduced to six, and in 1986, it will rise to seven. Initially, licences were granted for one year, but a revision of regulations in 1983 enabled the Authority to issue licences for up to seven years.

The Authority does not have the power to acquire fruit in its own right, although this power could be acquired by referendum if a substantial majority of producers supported such a move.

The original Kiwifruit Export Promotion Committee became one of the several committees operating under the Authority. Its scope was later widened, and it was renamed the Kiwifruit Marketing Planning Committee. It concentrates on promotion, and sets sales targets based on market research information.

The Authority and the Kiwifruit Exporters' Association established panels of agents in the main markets who, although operating independently, did so within a marketing plan. These agents report on trends in their market, the support needed to achieve sales targets, and realistic price levels. This information is passed back to the Authority and to producers. The Authority then arranges the promotion and other services necessary to achieve target price and offtake levels, while producers are expected to supply the right product handled in the right way to exporters. Exporters return an individual pool price to their own suppliers. Currently, different exporters' pool prices appear to be very similar (New Zealand Kiwifruit Authority, 1984).

Recent concern has emerged over excessive dependence on the West German market. As a result of this, the Authority has encouraged diversification away from this market. It has done this by imposing maximum proportions of the crop which can go to larger markets such as Germany

and Japan, and minimum proportions which must go to certain other markets, such as North America and 'developing markets' (Brash, 1985). Promotional budgets have also been skewed towards new markets for the same purpose.

One of the reasons for establishing the Authority was concern with poor quality fruit being sold on the export market. It was felt that this lowered wholesale prices and, consequently returns to growers producing high-quality fruit. When the Authority was set up, it was envisaged that it would have the power to control the quality of export fruit, but this provision was not made explicit until the 1983 revision of the Regulations.

However, with increasing overseas production poorer quality fruit is once again finding its way onto export markets, with the result, the Authority believes, of depressing the price of quality fruit which does not appear to be well-differentiated. With this in mind, the Authority is seeking the co-operation of overseas producer groups, while at the same time investigating ways in which the New Zealand product can be differentiated from that of its rivals.

The Authority is also involved in production research aimed at reducing production costs, and is informally involved in product development through participation in processing activities.

2.7.2 Economic Policies of Kiwifruit Agencies

The sole economic policy conducted by the Kiwifruit Export Promotion Committee which operated prior to 1977, was demand-shifting through generic promotion activities. Although it had no power to do so, it attempted to become involved in demand segmentation policies through its recommendations on grading, quality, packaging and storage.

The New Zealand Kiwifruit Authority, which was given statutory authority in 1977, continued these demand-shifting policies. Its other major policy appears to incorporate various aspects of market segmentation. For example, the primary objective of licensing is to facilitate a coordinated industry-style operation in the market place (New Zealand Kiwifruit Authority, 1984), and recently the Authority has encouraged market diversification by placing restrictions on certain markets. Ancillary activities which support demand segmentation include grading, packaging and participation in product development through processing. The Authority also conducts a supply-shifting policy through its involvement in research aimed at reducing production costs, and co-ordination of transport.

These economic policies are summarized in Figure 2.7.

2.7.3 Objectives of Kiwifruit Agencies

Through its activities, it appears that the objective of the Kiwifruit Export Promotion Committee was to

	KIWIFRUIT PROMOTION EXPORT COMMITTEE (1970-1977)	NEW ZEALAND KIWIFRUIT AUTHORITY (1977-1985)
<u>OBJECTIVES</u>	INCREASE GROWERS' RETURNS	INCREASE GROWERS' RETURNS
<u>EFFECTIVE POWERS</u>	TO DISBURSE VOLUNTARY LEVY FUNDS THROUGH PROMOTION	POWER TO INDIRECTLY CONTROL PRODUCT FLOW POWER TO ENFORCE STANDARDS POWER TO LEVY AND DISBURSE FUNDS
<u>ECONOMIC POLICIES</u>	DEMAND-SHIFTING (PROMOTION) DEMAND SEGMENTATION (GRADING, QUALITY, PACKAGING, STORAGE)	DEMAND SHIFTING (PROMOTION) MARKET DIVERSIFICATION DEMAND SEGMENTATION (LICENSING OF EXPORTERS, GRADING, PACKAGING, PROCESSING) SUPPLY SHIFTING (RESEARCH, CO-ORDINATED SHIPPING)

FIGURE 2.7: STATUTORY MARKETING IN NEW ZEALAND'S KIWIFRUIT INDUSTRY

maintain or increase returns to those growers contributing to levy funds.

The concern which led to the setting-up of the New Zealand Kiwifruit Authority, and the policies in which it engages, suggests that its primary objective was also to increase producer returns. The perceived aims of the licensing procedure also supports this objective.

2.7.4 Effective Powers of Kiwifruit Agencies

The Kiwifruit Export Promotion Committee had the power to disburse levy funds for promotion purposes as it saw fit. However, it had no statutory authority to levy all growers and exporters of kiwifruit, and was, consequently, plagued by free-rider problems.

The New Zealand Kiwifruit Authority, which has operated since 1977, has indirect power to control the flow of product onto markets through its licensing function and industry marketing plan. In addition, it has the power to levy and to disburse levy funds through promotion and research activities, and to enforce quality standards through grading and packaging.

2.8 CONCLUSIONS

2.8.1 The Evolution of Market Segmentation Policies

Statutory authorities in the four long-established industries were set up in the early 1920's, following a fall in prices after the First World War. A perusal of Figures 2.3 to 2.6 shows a broad common pattern in the

evolution of the economic policies pursued by these institutions.

The initial emphasis was on raising producer returns or stabilizing prices through demand and supply-shifting policies, and price-fixing and stabilizing schemes. For example, demand-shifting policies through promotion activities were initiated in the dairy, meat and wool industries prior to World War Two. Supply-shifting policies were instigated in the same period over the four industries through research, by reducing transport, processing and marketing costs, by a watch-dog role on costs in the meat industry, and by co-ordinated shipping arrangements. Minor demand segmentation was facilitated through grading schemes.

Various guaranteed pricing schemes came into operation in all four industries at one stage or another prior to 1954. These schemes were designed to either fix or to stabilize prices, or to achieve a combination of both, and they involved price pooling arrangements for producers. In addition, short-term price stabilization was facilitated by co-ordinated shipping in the dairy and meat industries, and by auction scheduling in the wool industry.

After World War Two, concern began to mount about the dependence on the United Kingdom market. This led to supply diversion and market diversification programs, which were in operation in the dairy, meat and pipfruit industries by the 1960's. These programs were forerunners

to the more sophisticated marketing policies eventually adopted in all industries.

By the late 1970's, most industries faced a depressed international trading environment with over-supply, a build-up of stocks, and restricted access to traditional markets. In response to this situation, the dairy, meat and pipfruit industries have instigated sophisticated market segmentation policies aimed at exploiting available market niches. Despite being structured differently to these other industries, the wool industry is also moving in this direction with product development and promotion aimed at specific market segments.

The history of the kiwifruit industry differs to that of the other major industries considered. It became established in the 1970's, and the New Zealand Kiwifruit Authority was set up in 1977 in response to a perceived threat of falling prices. Unlike the other institutions considered, this agency moved directly into demand management policies, by attempting to shift demand through promotion, and to segment the market through the operation of its exporter licensing arrangements.

In conclusion, therefore, there appears to be a definite trend towards the use of market segmentation policies by agencies operating in the five major agricultural exporting industries. In the dairy, meat and pipfruit industries, these are directly pursued by the Boards themselves, whereas in the kiwifruit industry, an attempt appears to be made to achieve them in a more indirect manner through the licensing, and consequent

direction, of exporters. Although the current situation in the meat industry is uncertain, it seems likely that the Board may continue to allocate rights to operate in certain markets or that companies may pursue this policy voluntarily, thereby retaining the ability of the industry to practise the market segmentation policies which the Board has developed. The wool industry, on the other hand, has made only minor moves towards market segmentation policies through product development and associated promotion by the New Zealand Wool Board and by the International Wool Secretariat.

2.8.2 Representative Objectives of Statutory Agencies

From an examination of Figures 2.3 to 2.7, it becomes obvious that all statutory agencies have a producer orientation, and that the long-term objectives of such institutions operating in the dairy, meat and wool industries are to increase returns to producers, to ensure equity among producers in receiving these returns, and to stabilize prices. However, the equity consideration is a relatively recent concern in the meat industry, and it is not certain that it will remain so, and price stability is a recent objective in the pipfruit industry. The kiwifruit industry appears to be primarily concerned with increasing producer returns, and does not seem to place any direct emphasis on price stability or equity at this stage. The wool industry tends to differ from other industries considered, since it does not emphasize this

equity objective. However, it stresses stabilizing prices and increasing producer returns.

In all the industries considered, these long-term objectives appear to be relatively stable, although the current structure in the meat industry is unstable, which impinges on the stability of the equity objective in this industry.

In the dairy, meat, pipfruit and kiwifruit industries, increasing producer returns appears to be the most important objective, with the principle of an equitable distribution of returns well-established in the dairy and pipfruit industries, although uncertain in the meat industry. Price stabilization appears to be either a secondary objective, or not considered to be of importance at all. However, the wool industry diverges from this pattern, since in this case, price stabilization is considered to be the primary objective.

In conclusion, therefore, a representative set of objectives for statutory agencies operating in New Zealand's major agricultural exporting industries would appear to be to increase producer returns, to ensure equity among producers, and to stabilize producer prices. Of these objectives increasing producer returns is most important, with price stabilization being a secondary objective. These objectives and their relative importance are sufficiently robust to be considered as reasonably representative of all industries studied except wool, where any assessment of market segmentation policies based on these representative objectives must be interpreted

with caution. Since many emerging industries tend to model themselves on these major industries, it seems reasonable to assume that the above objectives and their relative emphasis would be applicable in these cases also.

2.8.3 Representative Effective Powers of Statutory Agencies

The effective powers of statutory agencies operating in the dairy and pipfruit industries, and until recently, the meat industry, have been quite considerable. In these cases, such institutions can acquire and market all product, thereby giving them control over the volume of product which can be directed to particular destinations or end uses. In effect, they have virtual control over the entire marketing process, including handling, pooling, transportation, storage, processing and packaging. They have the power to levy producers, and to disburse these levy funds for the purpose of promotion and research.

In the above cases, either the agency itself, as in the meat industry prior to the 1985/86 season, or associated statutory bodies, as with dairy and pipfruit, set prices to be returned to producers. While these prices may be based on a number of criteria in the short-term, in the longer term they are linked to market realizations, thereby, in effect, returning a pooled price to suppliers.

In none of the cases considered, do statutory agencies have any power to control the level of industry output.

Unlike the above agencies, the New Zealand Kiwifruit Authority has no power to acquire product. However, it can indirectly influence the flow of product through the system of licensing exporters. It also has power to levy and to use such funds for promotion and research. In this industry also, producers receive pool prices from individual exporters, and it appears that such prices are similar.

Once again, the wool industry appears to be the exception. Despite having the power to acquire the wool clip, the New Zealand Wool Board chooses not to utilize this power. Instead, it trades when this is necessary to stabilize producer prices. As with the other agencies considered, it has the power to levy for promotion and research purposes. In addition, it has the authority to negotiate cost reductions on behalf of the industry at various stages of the marketing process.

The effective powers considered above can be considered quite stable for the dairy and pipfruit industries, where they have remained unchanged for over fifty years. However, in the case of the meat industry, the situation is uncertain. Although the Board recently had effective control of the entire marketing procedure, it remains to be seen which of these powers it retains and whether any of them are effectively disbursed to the private companies. At the moment, the powers of the Board cannot be considered stable. On the other hand, the respective powers of agencies in the wool and kiwifruit industries can be considered reasonably stable.

For the purpose of assessing market segmentation policies, a representative agency could be considered to have the power to direct specified volumes of product to specific destinations or end uses, and to return a pooled price to producers. It could also be considered to have the necessary power to acquire funds for promotion or product development, where this forms a necessary part of a market segmentation policy. However, it would not have the power to control the level of industry output.

Such representative powers are applicable to the dairy, pipfruit and kiwifruit industries, although in the kiwifruit industry, the power to control the flow of product to specific destinations or end uses is an indirect one, and not a direct result of product acquisition as in other industries. These powers characterized the meat industry until recently, although the current marketing structure is too unstable to determine whether it will effectively retain these powers in future. However, to consider such powers as representative of the wool industry presupposes radical industry reorganization, and therefore, must be interpreted with caution. Once again, it could be assumed that emerging industries may attempt to acquire the above representative powers, given their propensity to emulate existing industries.

CHAPTER 3

A FRAMEWORK FOR EVALUATING MARKET SEGMENTATION POLICIES

3.1 INTRODUCTION

The increasing importance of market segmentation as an economic policy practised by institutions operating in New Zealand's major agricultural exporting industries was established in the previous Chapter. In addition, a representative structure and set of objectives were developed for an institution which would typically operate such policies.

The objective of this Chapter is to establish a framework for evaluating market segmentation policies given this institutional structure and set of objectives. To this end, Section 3.2 expresses a market segmentation strategy in terms of an appropriate economic model. In Section 3.3, criteria for evaluating such a strategy are then considered, and an appropriate concept for measuring producer returns is discussed in Section 3.4. Section 3.5 critically reviews the relevant literature on market segmentation by agricultural marketing institutions, and finally, Section 3.6 outlines how this framework will be further developed in following Chapters.

3.2 AN ECONOMIC MODEL OF MARKET SEGMENTATION

3.2.1 Market Segmentation and Marketing Management

In the previous Chapter, the term 'market segmentation' was used to describe the practice by marketing institutions of seeking out and exploiting market niches for their product. Indications that such institutions were moving towards this type of policy were evident in their attempts to diversify markets, and to gear promotion activities and product development to specific market segments. The change in policy emphasis by these institutions reflects the increasing influence of the prescriptions of marketing management in agricultural marketing.

Watson (1983) attempts to chart this minor paradigm shift in the agricultural marketing literature. He notes the recognition by Breimyer (1973) of a 'market development' approach to agricultural marketing, where attempts are made to apply product development and promotion techniques developed for the industrial sector to problems of agricultural marketing. Bateman (1976) in his review of agricultural marketing literature, moves a stage further by attempting to emphasize the appropriateness of the 'marketing concept' to agricultural marketing by farmer organisations. A more revealing example of the growing acceptance of marketing management concepts is cited by Watson (1983) when he notes the shift in orientation between different editions of a standard agricultural marketing text. For example, Kohls and Uhl (1980) place less emphasis on agricultural price

analysis and government involvement in agricultural marketing, and more emphasis on the changing organisation of food markets and market development, than did their predecessors, Kohls and Downey (1972). Therefore, the managerial prescriptions of the marketing management perspective are becoming well-entrenched in both the principles and practices of agricultural marketing.

When the general term 'market segmentation' is used in the marketing management context, it tends to refer to the practices of segmenting a market, targeting specific market segments, and positioning product in target segments. This product positioning requires developing a marketing mix for each target segment where the marketing mix is a particular blend of controllable marketing variables (Kotler, 1984). These variables have been classified in various ways (Kotler, 1980), but the one most popularly used is that attributed to McCarthy (1960). He envisaged four variables in the marketing mix, these being product, price, place and promotion, commonly referred to as the "4 P's".

Writers concerned with the overlap between the economics and marketing management disciplines have noted an analogy between the discriminatory pricing model of the economics literature and the management principles for market segmentation (Watson, 1983; Nagle, 1984). That is, the managerial prescriptions for market segmentation can be partially embodied into an economic profit-maximization model which attempts to optimize marketing mix variables. Once a market has been segmented and target segment

identified, optimal prices and product levels for each target segment (or 'place') can be determined by applying the standard economic model of price discrimination.

3.2.2 Price Discrimination Defined

Price discrimination can be broadly defined as the sale (or purchase) of different units of a good or service at price differentials not directly corresponding to differences in supply cost (Scherer, 1980; Philips, 1983). Various techniques for doing this were originally identified by Cassidy (1946), and these have been extended and summarized by Machlup (1975).

When examining the theoretical basis for price discrimination, however, it has become customary to follow the classification of Pigou (1920), and to refer to price discrimination by a monopolist as being of a first, second or third degree type. Despite the misgivings of some writers as to the adequacy of this definition (Enke, 1964), it is the generally accepted classification, and will be followed in this study.

First-degree, or perfect, price discrimination occurs when each unit of a good or service is sold at its reservation price; that is, the discriminator appropriates all consumers' surplus as producers' surplus. Second-degree price discrimination is an imperfect variant of this. In this case, the demand curve is partitioned into a number of blocks, and a particular reservation price charged for each block. Therefore, the greater the number of blocks, the higher the proportion of consumers'

surplus which is appropriated by the producer. In the limit, second-degree price discrimination approaches the first degree variant as the number of blocks increase, and their individual areas decrease.

Perfect (first-degree) price discrimination implies zero transaction costs on the part of the discriminating monopolist. However, when such transaction costs are positive, some monopolists might efficiently choose not to engage in perfect price discrimination (de Alessi, 1983). Pigou's case of third-degree price discrimination covers this more realistic situation. Under these circumstances, it is assumed that the seller can divide customers into two or more groups, each with its own continuous demand function. If a set of appropriate pre-conditions exist, then the supplier can charge different prices for the same product in different markets. It is this classification of its market which most appropriately describes the situation facing marketing institutions operating in New Zealand export markets, and will therefore be investigated further.

For this third-degree price discrimination to be successfully practised, the potentially discriminating firm must have some monopoly power; that is, it must face a downward-sloping demand curve in aggregate. It must also be in a position to segregate customers into market segments with differing price elasticities of demand at a common price, and an ability to constrain opportunities for arbitrage between these market segments. Robinson

(1933) and Philips (1983) discuss these conditions in greater detail.

3.2.3 The Third-Degree Monopolistic Price Discrimination Model

The profit-maximizing condition associated with a third-degree price-discriminating monopolist can be derived as follows.

Assume a single-product firm which operates in two markets, 1 and 2. It must therefore decide which output, Q , to produce, and how to allocate this output to markets 1 and 2 in order to maximize profit. Let

$$(3.1) \quad R = R_1(Q_1) + R_2(Q_2)$$

be the total revenue function of the firm, where R_i is the revenue function in the i th market, and Q_i is the output allocated to the i th market. Let

$$(3.2) \quad C = C(Q)$$

where C is the total cost function of the firm, and

$$(3.3) \quad Q = Q_1 + Q_2$$

Therefore, the profit function, π , may be written as

$$(3.4) \quad \pi = R_1(Q_1) + R_2(Q_2) - C(Q)$$

Associated with this profit function are the following first order partial derivatives

$$(3.5a) \quad \frac{\delta \pi}{\delta Q_1} = R_1'(Q_1) - C'(Q)$$

and

$$(3.5b) \quad \frac{\delta \pi}{\delta Q_2} = R_2'(Q_2) - C'(Q)$$

The first order conditions for profit-maximization are met when (3.5a) and (3.5b) are equated to zero. That is,

$$(3.6a) \quad R_1'(Q_1) = R_2'(Q_2) = C'(Q)$$

or, in more familiar terminology,

$$(3.6b) \quad MR_1 = MR_2 = MC$$

where MR_i is the marginal revenue in the i th market and MC is the marginal cost. For the appropriate second-order profit-maximizing conditions associated with this model, see Chiang (1974).

Therefore, the levels of output (or product), Q_1 and Q_2 , allocated to market segments (or place) 1 and 2 respectively, are chosen such that the resulting prices, P_1 and P_2 , lead to the equating of marginal revenue in each market segment with the marginal cost of the total profit-maximizing output, Q .

The comparative statics nature of the above model abstracts from the adjustment path of variables to their price-discriminating equilibrium values. Despite this, it

would be appropriate to use a modified form of the model as a basis for examining market segmentation policies by agricultural marketing institutions, since the major objective of this study is to determine the conditions under which such policies are likely to be successful for producers, rather than to trace through their dynamic path adjustments.

However, there are problems associated with an unmodified application of this model. The first problem arises because the standard theory of price discrimination outlined above refers to a profit-maximizing monopolist, who optimizes the total output decision. However, it was established in Chapter 2 that marketing institutions operating in typical New Zealand agricultural exporting industries have the power to allocate output to alternative destinations, but not the power to control the aggregate level of output.

A second problem is associated with the fact that the above model refers to the behaviour of a monopolist. However, New Zealand marketing institutions are not the sole suppliers in export markets for most of their products, and therefore, face the prospect of competitive responses to their pricing policies.

Another problem associated with this model concerns the fact that it does not explicitly consider promotion, which is the fourth variable in the marketing mix outlined above. The extent to which these issues have been addressed in the literature will be examined in Section 3.5 below.

3.3 EVALUATION CRITERIA

In the previous Chapter, a set of long-run objectives were determined for a statutory institution operating on behalf of producers in an agricultural exporting industry. These were to increase producer returns, to stabilize producer prices, and to ensure equity among producers. Of these objectives, increasing producer returns appears to be the most important, and would take precedence over other objectives if any conflict arose. Therefore, market segmentation policies can be evaluated by determining the extent to which they achieve these three objectives.

The third objective of ensuring equity among producers is automatically met by the payment procedures of a typical marketing institution, since it allocates output to alternative market segments and returns suppliers a pooled price, which is the weighted average of returns from these markets. The second objective of stabilized producer prices can be evaluated under comparative statics analysis by comparing producer price variability under competitive pricing with that under discriminatory pricing. Likewise, the first objective of increasing producer returns can be evaluated by comparing the returns which producers would have received under a competitive product allocation to alternative market segments with those accruing under product allocation with discriminatory pricing.

3.4 THE CONCEPT OF PRODUCER SURPLUS

In order to evaluate the relative returns to producers under alternative pricing schemes, a measure of such returns must be adopted. The most common measure used to determine the collective returns to a producer group resulting from policies instituted by government or some other agency, is producer surplus. This producer surplus is given by the area above the industry supply curve and below the product price, and is assumed to represent economic rent accruing to suppliers after the opportunity cost of resources used has been taken into account.

Therefore, for an inverse industry supply curve

$$(3.7) \quad P = S(Q)$$

where P and Q are product price and output respectively, then producer surplus, PS , is given by

$$(3.8) \quad PS = PQ - \int S(Q) dQ$$

However, there is some dissension in the literature as to how this measure of producer surplus should be interpreted. Such concerns focus on the short-run or long-run nature of the product supply curve, and hence on the rent concepts implied by the area traditionally referred to as 'producer surplus'.

In the short-run, cases can be identified where the area above the industry supply curve and below the price line can be, in a perfectly competitive industry, unambiguously interpreted as a measure of rent. This occurs when the output of one factor is fixed, and the

prices of all variable factors are fixed; that is, their supplies are perfectly elastic to the industry (Currie, et al, 1971). In the case where land is the fixed factor, then this measure of producer surplus is equivalent to Ricardian rent, and where capital is fixed, it is equivalent to Marshallian quasi-rent (Mishan, 1982). In both cases, surplus accrues to owners of firms in their production and sale of the product, but the source of this rent is the ownership of a fixed factor of production.

However, the situation faced with the long-run product supply curve is not quite so clear. In the long-run, the product supply curve is the locus of minimum average costs for each firm in the industry. In this case, no excess profits are made, and therefore, the interpretation of producer surplus as a rent payment is not so obvious (Mishan, 1968). However, these average costs include factor payments which are surpluses, and it then becomes possible to conclude that the area above the supply curve is a meaningful rent measure when the supply curve, in addition to being an average cost curve including rents, is a marginal cost curve excluding rent (Currie, et al, 1971).

These conditions will be met in two circumstances. The first occurs when the long-run supply of land is fixed, but other factors are available at constant prices. The application of these variable factors to the fixed factor, land, leads to diminishing returns and an upward sloping supply curve, which may be interpreted as a long-run average cost curve including rent to land, but a

long-run marginal cost curve excluding rent to land. The second situation arises where one necessary factor has some inelasticity in supply, while other factors are elastic with respect to their supply. In this case, also, the long-run industry supply curve will be an average cost curve including rents, and a marginal cost curve excluding rents, with these rents amounting to the excess payments made for intramarginal units of the inelastic factor (Currie, et al, 1971).

However, problems of interpretation arise where there are two variable inputs, neither of which is elastic with respect to supply. In this case, it can be shown that any increase in producer surplus is made up of gains to the intensive variable factor, less losses to the extensive variable factor (Mishan, 1982), and therefore, the supply curve cannot be interpreted as a marginal cost curve excluding factor rents (Currie, et al, 1971). Hence, the economic significance of the concept is questionable, since producer surplus cannot be taken as a gain to either factor, nor to both factors taken together. Shepherd (1970) disputes this, and argues that the relevant area represents excess earnings over the amount necessary to keep factors in their present occupation, and therefore, provides an accurate measure of a Paretian rent concept, where the product supply curve represents a marginal cost curve excluding rent. However, he does not address the question of whether this aggregate producer rent represents a net calculation of the sum of factor losses and gains, and leaves unchallenged the notion that an

aggregate measure such as this is a meaningless concept with respect to returns to factor classes.

In summary, therefore, the concept of producer surplus provides a meaningful measure of returns to producers in a perfectly competitive industry in both the short and long-run, when one factor is in fixed or relatively inelastic supply to the industry, while the provision of other factors is elastic. In this study, it is assumed that the factor land is in relatively inelastic supply, and that the price of labour and capital is fixed over any increased output range induced by the introduction of market segmentation policies. Such assumptions appear reasonable, and are obviously implicit in the wide range of agricultural studies which utilize the producer surplus concept for policy evaluation.

3.5 REVIEW OF THE LITERATURE

3.5.1 Introduction

In Section 3.3, the basic model of third-degree monopolistic price discrimination was described. However, a number of problems were identified with respect to an unmodified application of this model to an examination of market segmentation strategies by agricultural marketing institutions.

Firstly, marketing agencies operating in typical New Zealand agricultural exporting industries do not have the power to control output. Therefore, when a marketing institution practises price discrimination by diverting

output between market segments, and returns a pooled price to suppliers, then some sort of supply response can be expected, since this pooled price will be greater than the price received under a competitive allocation of product between market segments.

Secondly, New Zealand marketing institutions are not the sole suppliers of their products in most export markets. Therefore, their pricing policies will generate a competitive response from alternative suppliers which must be taken into account when formulating these policies.

The third problem arises from the fact that this simple discriminatory pricing model does not consider promotion as an explicit variable. Conceptually, two promotion effects could occur which would influence optimal segmentation policies. Firstly, product demand could be increased in particular market segments, and secondly, a New Zealand product could be differentiated from that of a competitor.

The extent to which these three issues have been incorporated into models of price discrimination in the agricultural economics literature will now be reviewed.

3.5.2 Own Supply Response to Discriminatory Pricing

A number of authors have recognized that a supply response will occur when a marketing institution price discriminates between market segments and returns a pooled price to its suppliers. For example, in his study of supply diversion strategies by New Zealand's Apple and

Pear Marketing Board, Rae (1978) noted that, through discriminatory pricing, producers who supply the Board will be encouraged to expand production. Similarly, in her study of price-discriminating activity by the New Zealand Dairy Board between the butter and cheese segments of the United Kingdom market, Veeman (1972) cautions that the Board's activities will be constrained by the fact that it has no control over entry into the industry by suppliers, nor over the volume of milk produced. Likewise, in his study of controlled allocation schemes for the United States apple crop, Piggott (1976) pointed out that increasing the net revenue for each year's apple crop may have the long-run effect of attracting resources into the industry. However, although these authors recognized that such a response could occur, they did not include it in their studies.

A further group of studies, however, do incorporate a supply response into their analyses. Freebairn and Gruen (1977) considered Australian beef export diversification schemes where non-uniform prices were charged across different markets, and a supply response was included. Reeves and Longmire (1982) did likewise when analyzing voluntary restraint in one of their export markets. Similarly, Banks and Mauldon (1966), in their analysis of the two-price scheme in the West Australian egg market, incorporated the response which results from returning suppliers an equalized price. Baritelle and Price (1974) also included a supply response in their analysis of price discrimination in the Washington State apple industry.

The manner in which this supply response has been incorporated into a price discrimination framework tends to vary between studies. For example, Baritelle and Price (1974) estimated a supply response function and demand functions for the alternative fresh and process markets. Given price-discriminating behaviour, simulations were then run for a time horizon of ten years, and the results noted.

However, other authors have adopted a more analytical approach, utilizing combinations of graphical, geometric and algebraic techniques to derive results. Banks and Mauldon (1966) graphically illustrate their two-price scheme which they then translate into an algebraic model, thereby allowing them to make estimates of the effects of a price discrimination policy. However, since their model only includes two markets, one of which has specialized demand conditions, it is of limited applicability to the more generalized problem of price discrimination among a number of export markets.

Gardner (1983) makes the observation that price discrimination increases the farm price received for any given quantity of produce marketed, and therefore, has the same effect as a rightward shift in demand. This effect has been noted in other studies. For example, although Reeves and Longmire (1982) are not considering an optimal price-discriminating situation to the extent that they exercise voluntary restraint in one export market, while allocating the excess output to alternative export markets, they graphically illustrate the effective

rightward shift in demand resulting from this policy. They show a new price-quantity or 'demand' relationship which lies above a linear aggregate excess demand curve, and which asymptotically approaches this latter curve in the limit. Sieper (1982), in his illustration of price discrimination between two export markets facing downward-sloping linear demands, shows an average (price-discriminating) export revenue curve which appears to have similar properties. Therefore, it would be desirable to develop a model which would allow the characteristics of this price-discriminating average revenue curve and its relationship to the non-discriminating aggregate export demand curve to be investigated.

In addition to noting that a supply response will occur under these conditions, some authors have commented on the effect of such a response on resource use and on producer returns. Concern has been expressed that such policies induce sub-optimal resource use. For example, Banks and Mauldon (1966) note that if producers received only the export price for their marginal production, a smaller number of eggs would have been produced. They therefore concluded that waste results from failing to restrict production to an output level where the cost of additional supply is equal to the export price. In their analysis of export diversification, Freebairn and Gruen (1977) reach a similar conclusion, commenting that production expansion effects are not likely to be beneficial in the sense that more resources would be drawn

from relatively low cost industries than from highly protected ones. The New Zealand Treasury (1984) has recently echoed this view, noting that problems are caused when the averaging of prices paid to producers calls forth production which costs more to achieve than it earns in the lowest paying 'residual' markets.

This supply response will also affect producer returns. Baritelle and Price (1974) expressed concern that initially higher returns would increase supplies, which would then reduce returns to their original level. However, the results of their simulations suggested that the returns to growers were much higher under a price-discriminating policy than under a non-discriminating one, despite the fact that a substantial supply response had occurred. Weisenborn (1969) in his study of the allocation of Florida orange production also noted that when supply exceeded his estimated optimum, some form of abandonment or internal supply control would be necessary, thereby implying that a supply response could decrease producer returns.

Other researchers have tried to identify factors which influence the magnitude of producer returns under price discrimination. Gardner (1983) notes that as long as some farmer-owned production inputs are not perfectly elastic in supply, then producer surplus will increase under price discrimination without supply control. Reeves and Longmire (1982) concluded that export control schemes stimulate production, thereby leading to increased supplies in uncontrolled markets, which lowers the average

price. They noted that the extent to which exports in uncontrolled markets increase, and to which prices decrease, depends on the elasticities of export demand and export supply. Although Reeves and Longmire (1982) analyze a voluntary restraint agreement rather than a price-discriminating situation, such agreements have discriminatory elements, and their effects are therefore of relevance.

In summary, therefore, the issue of an own supply response to discriminatory pricing is well recognised in the literature. Some studies have included it in their analyses, although these have tended to be rather specialized, and consequently, are limited in their application. It appears that discriminatory pricing effectively leads to a rightward shift in aggregate demand; although factors influencing the extent of such a shift do not appear to have been investigated. The magnitude of the supply response, and its effect on producer returns, appears to be influenced by the price elasticity of supply and the price elasticity of export demand.

3.5.3 Competitive Supply Response to Discriminatory Pricing

The possibility that discriminatory pricing policies may generate a competitive response from alternative suppliers has concerned researchers investigating such policies. Rae (1978) notes that discriminatory pricing by the New Zealand Apple and Pear Marketing Board will

encourage direct sales suppliers to expand production in response to increased domestic prices.

Other authors have cautioned that 'optimal' allocation policies are constrained by potential competition. For example, Banks and Mauldoon (1966) point out that domestic egg prices cannot be set at a higher level than the price which will induce eggs on to the local market from alternative supply sources. Edwards (1970), in his determination of an optimal allocation of New Zealand lamb exports between the UK and USA, noted that his results would be invalidated by reactions from other suppliers to these markets. Similarly, Weisenborn (1969) noted that the increased sales of fresh Florida oranges suggested by his model seemed unreasonable, given the competitive relationships existing between orange-producing States. Freebairn and Gruen (1977) go further than these authors, and suggest that it may be necessary to reach a high degree of co-operation with alternative exporting countries to counter this problem of a competitive supply response.

Some authors who have recognised that marketing institutions may not be monopolists in their export markets have attempted to account for this in their analyses. Edwards (1970) and Veeman (1972) both discount the price elasticity of demand facing such an institution in a particular market by the market share which they hold in that market. This effectively means that such agencies face more price elastic demands for their product than suggested by the market elasticity.

However, as Edwards (1970) notes, while the above formulation acknowledges the presence of competitors, it assumes that these competitors will not react. Abel (1966), in a general study of price discrimination in world trade for agricultural products, commented on demand elasticities faced by exporters in importing countries. He noted that the price responsiveness of this import demand relationship depends on the price responsiveness of both demand and supply in these importing countries. However, he did not indicate how these demand elasticities facing exporters were altered by these factors.

To summarize, a competitive supply response has been acknowledged by writers concerned with discriminatory pricing policies. It appears that the presence of competitors, and their price responses, influence the price elasticity of demand faced by exporters. Therefore, factors such as market share and competitors' price elasticities of demand and supply influence the magnitude of the competitive supply response. However, these features do not appear to have been explicitly modelled in the context of discriminatory pricing.

3.5.4 Promotion Effects

The standard economic model of third-degree price discrimination does not explicitly consider promotion, which is the fourth variable in the marketing mix. That is, the model does not optimize the promotion decision.

However, the general theory of optimal advertising has attracted a great deal of attention in the economics and

marketing literature. Schmalensee (1972), Lambin (1976) and Koutsoyiannis (1982) review such research. Some of these reported studies include marketing mix optimization models, where a promotion variable has been incorporated (Claycamp and Massy, 1968; Lambin, 1976).

However, despite this plethora of optimal promotion literature, much of it, including the optimal marketing mix variants, is not directly applicable to agricultural industries. The reason for this is that such literature tends to concentrate on market structures with monopolistic or oligopolistic supply features, where optimal output decisions are determined in conjunction with optimal pricing and advertising decisions. Such market structures do not characterize the supply-side of agricultural markets, and therefore, an unmodified application of the above models to the agricultural situation is not possible.

Despite this abundance of research dealing with imperfect market structures, the theory of optimal advertising in agricultural markets has been relatively neglected (Strak, 1983). Research which has been reported in the agricultural economics literature tends to focus on the Dorfman-Steiner theorem of optimal advertising and its extensions. Dorfman and Steiner (1954) optimized the advertising and product-price decisions for a monopolist operating in one market. This theory has been extended in its static form to include the advertising behaviour of a price-discriminating monopolist in two markets (de Boer, 1977), and to incorporate oligopolistic market structures

(Strak and Gill, 1983) and market structures with competitive supply conditions (Nerlove and Waugh, 1961).

The de Boer (1977) study was specifically concerned with two-price schemes. By utilizing optimal discriminatory pricing conditions and the Dorfman-Steiner theorem of optimal advertising intensity, he was able to deduce relative optimal advertising intensities in both markets for a profit-maximizing, price-discriminating monopolist. However, as he noted, his conclusions imply that industries have a significant degree of supply control.

Nerlove and Waugh (1961) included a competitive supply response to promotion in their study. They derived an optimal advertising decision rule for institutions which sought to maximize producer returns, and which were operating in industries without supply control, and therefore, the product-price decision was not optimized. Although the Nerlove-Waugh model has been criticized because of its simplistic single product, single market assumptions (Tisdell, 1976; Strak and Gill, 1983), it was a significant advance for the literature on advertising in agricultural industries, because it incorporated the appropriate supply-side features.

Attempts have been made to relax the restrictive assumptions noted above which are implicit in the Nerlove-Waugh theorem. For example, Thompson and Eiler (1977) extended the theorem to a managed domestic market of two sectors. However, only one of these market segments was subject to advertising. Other authors

directly considered the allocation of advertising to multiple markets or products (McClelland et al, 1971; Tisdell, 1976), where consumer sales rather than producer returns were maximized, subject to a fixed advertising budget.

However, although these latter studies relax the Nerlove-Waugh product and market assumptions, in order to do so, they introduce an alternative set of restrictive assumptions. For example, in both cases, price is implicitly assumed to be fixed and supply to be elastic. In addition, advertising levels are not necessarily optimal, but are merely allocated in an optimal manner. Therefore, despite their extension to a multiple market segment situation, the prescriptions of such research are not applicable to the present study, since they abstract from the product-price response to promotion activity.

The literature indicates that the problem of how to determine the optimal level of advertising in multiple markets in which prices are linked either competitively or through discriminatory pricing, has not been adequately investigated, particularly where supply is uncontrolled. Consequently, there appears to be scope for investigating this issue further by deriving appropriate decision rules for promotion activity in multiple market segments for discriminatory pricing and for the benchmark competitive pricing where supply is uncontrolled in both cases.

While such decision rules yield valuable information on the optimal direction of promotion effort, they would

not, however, give any indication of the relative magnitude of producer gains from promotion activity in alternative market segments. Therefore, there would appear to be further scope for investigating the influence of advertising on producer returns.

Conceptually, two promotion effects can be identified. The first increases product demand at all prices in particular market segments while the second differentiates product from that of a competitor in a particular market segment. The distinction between these two promotion effects, and their intermediate variants have been investigated by Schultz and Wittink (1976) and May (1977).

Although the first effect is well recognized in the literature as a specific type of generic advertising effect, there does not appear to be a great deal of comment on how such advertising in individual market segments influences producer returns.

With respect to the second effect, when a product can be differentiated from that of a competitor through advertising, demand for this product becomes less elastic than it would otherwise be (de Boer, 1977; May, 1977). This is seen to be desirable in terms of producer returns, since it allows a firm or institution with the ability to manipulate demand to extract greater monopoly rent from consumers than would have been possible otherwise (Parish, 1963). However, it has been noted that where price stability is a prime objective, then farmers may benefit from promotion which makes demand more price elastic,

since this will reduce price instability resulting from output variations (Waugh, 1959).

There appear to be no studies reported in the agricultural economics literature which examine the gains to producers from differentiating demand from that of competitors, where pricing is discriminatory. However, when attempting to derive the export demand for a product facing New Zealand in a particular market, Veeman (1972) and Edwards (1970) note that discounting the market price elasticity of demand by market share assumes that New Zealand's product and that of its competitors are perfect substitutes. Product differentiation would, therefore, make this export demand less elastic. It would also decrease any competitive supply response, which would further influence this demand elasticity. Abel (1966) notes that trade barriers in their various forms have a similar effect to product differentiation in reducing export demand elasticities.

In summary, there appears to have been very little research in the agricultural economics literature on theoretical aspects of increasing demand in particular market segments through promotion, or of differentiating demand from that of competitors. There seems to be scope for investigating optimal product promotion in multiple markets under appropriate supply conditions, and for determining factors which influence the magnitude of returns to producers from promotion which increases demand in alternative market segments. With respect to the promotion effect which differentiates a product from that

of a competitor and thereby alters the export demand elasticity facing a marketing agency for its product, it would appear to be useful to identify the conditions under which such a strategy would be successful, given discriminatory pricing.

3.6 CONCLUSION

In this Chapter, the economic model of third-degree price discrimination was investigated as an appropriate framework for evaluating market segmentation policies by agricultural marketing agencies with those institutional features outlined in the previous Chapter.

An own supply response to discriminatory pricing has been recognized in the literature, although it has not been analytically modelled for multiple export markets. Similarly, although a competitive supply response has been acknowledged, it has not been incorporated into discriminatory pricing models in any comprehensive sense. Although both of these issues have been dealt with at varying levels of sophistication by a number of authors, there appear to be no studies which simultaneously include both types of supply response to discriminatory pricing by marketing agencies. Consequently, a model will be developed in Chapter 4 which incorporates these features.

The theory of optimal promotion in multiple market segments without supply control has not been adequately investigated in the literature under either competitive or discriminatory pricing. Therefore, there is little

indication as to which market segments relative advertising effort should be directed under these alternative pricing regimes. In addition, there appears to have been little research on factors which influence the actual magnitude of producer returns when promotion of either a demand-shifting or product-differentiating nature is undertaken. Therefore, Chapter 5 considers such promotional issues.

CHAPTER 4

A THEORETICAL MODEL OF PRICE DISCRIMINATION WITHOUT SUPPLY CONTROL

4.1 INTRODUCTION

With the exception of promotion, the marketing mix components of any market segmentation policy are encapsulated in the standard economic model of third-degree price discrimination. However, it was established in the previous Chapter that this model, in an unmodified form, does not consider an own supply response to discriminatory pricing, nor does it include any competitive supply response which might result from such a pricing policy.

Therefore, the objective of this Chapter is to develop a general model of gains from price discrimination which incorporates these features. Such a model can then be used to evaluate whether market segmentation policies assist in achieving the primary objectives of agricultural marketing institutions, these being to increase producer returns and reduce price variability.

Consequently, in Section 4.2, a basic competitive pricing model is developed, against which a discriminatory pricing model can then be evaluated. Section 4.3 establishes the condition which maximizes returns from price discrimination in multiple markets without supply

control. In Section 4.4, a general algebraic price discrimination model which incorporates this maximizing condition and which calculates immediate revenue returns to producers is developed. Section 4.5 then considers an own supply response to discriminatory pricing, while Section 4.6 includes the corresponding competitive supply response. In Section 4.7, the increased returns to producers from adopting a discriminatory pricing policy are calculated, and finally, Section 4.8 summarizes important features of the model.

4.2 A COMPETITIVE PRICING MODEL WITH MULTIPLE MARKET SEGMENTS

Since this model is concerned with the relationship between output (product) and price in different market segments (place), let ceteris paribus conditions apply and allow the demand function in the i th market segment to be characterized by its inverse (price-dependent) form

$$(4.1a) \quad P_i = f(Q_i)$$

where P_i and Q_i are price and quantity, respectively, in market segment i .

To allow numerical examples of the model to be investigated, it is necessary to specify the functional form of this relationship. A linear function has been chosen for two reasons.

Firstly, when linear demand functions in individual market segments are summed, the result is a linear demand relationship in aggregate. A linear aggregate demand

curve such as this has been used by Reeves and Longmire (1982) and Sieper (1982) when considering models with discriminatory pricing elements. In both cases, their graphical illustrations imply a new average revenue curve after price discrimination which has specific functional properties. If linear aggregate demand properties are preserved in this model, then comparison with the above studies becomes one method of validating the model. A second reason for choosing linear demand functions is that this eases the burden of algebraic manipulation which would arise if more complex functional forms were adopted. This reduces potential sources of algebraic error and retains clarity in the model.

If it is intended to apply the model predictions to markets known to be characterized by non-linear demand relationships in market segments, then the model results will remain valid as long as changes in outputs and prices induced by applying discriminatory pricing to previously competitive markets are incremental. That is, only small changes from initial equilibrium values are involved. However, the use of linear relationships, and the above caveat on their use, are commonplace in agricultural studies (Wallace, 1962; Piggott, 1981).

Therefore, allow demand in the i th market segment to be more specifically represented by

$$(4.1b) \quad P_i = a_i - b_i Q_i; \quad a_i, b_i > 0$$

where P_i and Q_i are price and quantity, respectively, in market segment i .

As a result, the aggregate inverse demand curve becomes

$$(4.2) \quad P_a = a_a - b_a Q_a$$

where P_a is the competitive price

Q_a is the total quantity demanded

$$a_a = \frac{\sum_{i=1}^m \frac{a_i}{b_i}}{\sum_{i=1}^m \frac{1}{b_i}}$$

$$\text{and } b_a = \frac{1}{\sum_{i=1}^m \frac{1}{b_i}}$$

The derivations of a_a and b_a are shown in Appendix 1, as are all other proofs to equations presented in this Chapter.

All market segments may not be functional at all times. That is, some market segments may have no output demanded in them at certain prices. For example, for a given level of equilibrium output, Q_0 , the corresponding competitive price, P_0 , can be determined from equation (4.2). If n potential market segments are ranked such that $a_1 > a_2 > \dots > a_i > \dots > a_n$, then if $a_i > P_0$, i markets will remain functioning under the competitive price, P_0 . Let m denote the number of market segments which are functional at competitive equilibrium in the aggregate market. Therefore, equation (4.2) represents the segment of the linear aggregate demand curve for the m functional markets.

The supply side of the model can be similarly characterized in a linear inverse form by

$$(4.3) \quad P_s = c + d Q_s; \quad d > 0, \quad c < a_s$$

Competitive equilibrium is then determined when

$$(4.4) \quad P_d = P_s$$

At this point, the equilibrium price, P_0 , and output, Q_0 , are therefore given by

$$(4.5a) \quad P_0 = a_s - b_s \left[\frac{a_s - c}{b_s + d} \right]$$

and

$$(4.5b) \quad Q_0 = \frac{a_s - c}{b_s + d}$$

Output sold in the i th market segment, Q_{i0} , is determined by allowing $P_i = P_0$ in equation (4.1b), and then solving for the output level, Q_{i0} . The share of total output sold to market segment i under competitive pricing, s_{i0} , is then given by $\frac{Q_{i0}}{Q_0}$. Consequently, all output, Q_0 , is allocated to functional market segments, m . That is, $\sum_{i=1}^m Q_{i0} = Q_0$.

By restating equation (3.8) in Chapter 3 in a more specific form, producer surplus under competitive pricing, PS_c , is given by

$$(4.6) \quad PS_c = P_0 Q_0 - \int_x^{Q_0} (c + dQ) dQ$$

where $x = 0$ if $c \geq 0$

$$x = -\frac{c}{d} \text{ if } c < 0$$

This basic competitive pricing model with multiple markets segments may now be used as a benchmark to evaluate the discriminatory pricing model which will be constructed in later Sections.

4.3 MAXIMIZATION OF RETURNS UNDER PRICE DISCRIMINATION WITHOUT SUPPLY CONTROL

First-order profit maximization for a third-degree price-discriminating monopolist occurs when marginal revenues in individual markets are equal to each other and to the marginal cost of production. In general,

$$(4.7) \quad MR_1 = MR_2 = \dots = MR_i = \dots = MR_n = MC$$

where MR_i is marginal revenue in the i th market

and MC is marginal cost of production.

However, as noted in Chapter 3, agricultural marketing institutions typically have the power to allocate output to alternative destinations, but not the power to control the aggregate level of output. That is, the aggregate output decision may not necessarily be optimal, but will be determined by suppliers in response to the price which they receive. Therefore, given this situation, it is necessary to determine the condition which maximizes returns to producers.

A general measure of returns to producers, producer surplus, is given by the area under the price line and above the supply curve. For any given level of output, Q , this area is measured by total revenue, PQ , less the opportunity cost of resources used to produce this output. Therefore, given that this output, Q , has been produced,

and an opportunity cost of resources incurred, producer surplus will be maximized when PQ, or total revenue, is maximized.

If it is assumed that there are no costs associated with reallocating output among market segments, then a constrained maximization problem can be formulated as follows:

$$\text{Maximize TR} = \sum_{i=1}^m P_i Q_i$$

$$\text{subject to the constraint } \sum_{i=1}^m Q_i = Q$$

where TR is total revenue

Q is given output

and P_i and Q_i are price and output, respectively, in market segment i .

This problem can now be solved using the method of the Lagrange multiplier, and the producer surplus maximizing price and output levels determined.

To facilitate this, a two market segment case will be considered. By substituting equation (4.1b) into the maximization problem above, the Lagrangian function becomes

$$(4.8) \quad Z = (a_1 - b_1 Q_1) Q_1 + (a_2 - b_2 Q_2) Q_2 + \lambda (Q - Q_1 - Q_2)$$

The first order conditions associated with this now unconstrained objective function are

$$(4.9a) \quad \frac{\delta Z}{\delta Q_1} = a_1 - 2b_1 Q_1 - \lambda = 0$$

$$(4.9b) \quad \frac{\delta Z}{\delta Q_2} = a_2 - 2b_2 Q_2 - \lambda = 0$$

$$(4.9c) \quad \frac{\delta Z}{\delta \lambda} = Q - Q_1 - Q_2 = 0$$

By rearranging, this set of simultaneous equations can be transformed to matrix notation as follows.

$$(4.10) \quad \begin{bmatrix} 2b_1 & 0 & 1 \\ 0 & 2b_2 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ Q \end{bmatrix}$$

This can be solved for the vector $\{Q_1 \ Q_2 \ \lambda\}$ by applying Cramer's Rule.

This yields the Lagrange multiplier.

$$(4.11) \quad \lambda = a_a - 2b_a Q$$

Substituting equation (4.11) back into equations (4.9a) and (4.9b) and rearranging, gives

$$(4.12) \quad a_1 - 2b_1 Q_1 = a_2 - 2b_2 Q_2 = a_a - 2b_a Q$$

Now recall equation (4.1b)

$$(4.1b) \quad P_1 = a_1 - b_1 Q_1$$

Total revenue in this market segment, TR_1 , is calculated by multiplying this equation by output allocated to the market segment, Q_1 . The corresponding marginal revenue, MR_1 , is then derived by differentiating TR_1 with respect to Q_1 . This gives

$$(4.13a) \quad MR_1 = a_1 - 2b_1 Q_1$$

By similarly manipulating equation (4.2), marginal revenue in the aggregate market, MR_a , is given by

$$(4.13b) \quad MR_a = a_a - 2b_a Q_a$$

Therefore, equation (4.12) above is equivalent to

$$(4.14a) \quad MR_1 = MR_2 = MR_a$$

This can be generalized to

$$(4.14b) \quad MR_1 = MR_2 = \dots = MR_i = \dots = MR_n$$

where MR_i is marginal revenue in the i th market segment

and MR_n is marginal revenue in the aggregate market

Hence, this condition indicates that, if producer surplus is to be maximized, then any output produced must be allocated such that marginal revenues in each market segment are equated to each other and to aggregate marginal revenue. This is similar to the profit maximizing condition for a price-discriminating monopolist shown in equation (4.7) to the extent that marginal revenues in individual market segments are equated. However, the condition that they are, in turn, equated to marginal cost is not preserved. The functional form used in this model also allows the marginal revenues in individual market segments to be linked to marginal revenue in the aggregate market. As will be shown in later Sections, this model feature has a great deal of analytical value.

This Section has established the condition which maximizes returns from price discrimination in multiple markets without supply control. In the next Section, a general algebraic model of price discrimination will be developed which incorporates this maximizing condition and which calculates revenue gains from the pricing policy.

4.4 A GENERALIZED MODEL OF PRICE DISCRIMINATION

4.4.1 Optimal Quantities and Prices in Market Segments

The competitive pricing model with multiple market segments outlined in Section 4.2 will now be modified to incorporate discriminatory pricing. The optimal allocation of a given level of output to market segments is calculated, and the corresponding optimal prices in these segments are derived.

It was established in equation (4.12) above that producer surplus will be maximized when output produced is allocated such that marginal revenues in each market segment are equated to each other and to aggregate marginal revenue.

Recalling (4.13a), marginal revenue in the i th market segment, MR_i , was given by

$$(4.13a) \quad MR_i = a_i - 2b_i Q_i$$

Similarly, marginal revenue in the aggregate market was given by

$$(4.13b) \quad MR_a = a_a - 2b_a Q_a$$

Now, by equating (4.13a) and (4.13b), and rearranging terms, the optimal price-discriminating quantity in the i th market segment, Q_{i_d} , becomes

$$(4.15) \quad Q_{i_d} = \frac{2b_a Q_a + a_i - a_a}{2b_i}$$

where Q_a is the given output to be allocated among market segments

It is of interest to note that, in the two market segment case, optimal price-discriminating output allocations to market segments 1 and 2, would have been

determined when solving equation (4.10) for the vector $\{Q_1, Q_2, \lambda\}$ in the previous Section. However, both of these resulting outputs, Q_1 and Q_2 , were expressed in terms of the slopes and intercepts of each of the two markets. Therefore, they do not have the generality of equation (4.15) where output in a particular market segment, Q_{1i} , is expressed in terms of slopes and intercepts in that market segment and the aggregate market. This more general formula means that optimal output can be easily calculated where more than two market segments exist. Consequently, the alternative formulations are not presented.

The optimal price in market segment i , P_{1i} , can be determined by substituting (4.15) into (4.1b) and rearranging terms. This gives

$$(4.16) \quad P_{1i} = \frac{1}{2}(a_1 + a_2) - b_2 Q_a$$

where Q_a is the given output to be allocated among market segments

It was noted in the previous Chapter that some authors were attempting to relate returns from pricing with discriminatory elements to the magnitude of price elasticities of demand and supply (Reeves and Longmire, 1982; Gardner, 1983; Abel, 1966). Therefore, equations (4.15) and (4.16), and subsequent model equations, will be transformed to include these variables. Such a technique has previously been used by Piggott (1981).

Because demand equations in the model are linear, the slopes, b_i , and the intercepts, a_i , can be transformed using the following equations.

$$(4.17a) \quad a_i = P_a \left[1 + \frac{1}{n_i} \right]$$

$$(4.17b) \quad b_i = \frac{P_a}{Q_i} \cdot \frac{1}{n_i}$$

where P_a is the competitive price before discrimination

Q_i is the output allocated to market segment i before discrimination

and n_i is the price elasticity of demand (absolute value) at (P_a, Q_i)

Similarly, in the aggregate market,

$$(4.17c) \quad a_a = P_a \left[1 - \frac{1}{n_a} \right]$$

and

$$(4.17d) \quad b_a = \frac{P_a}{Q_a} \cdot \frac{1}{n_a}$$

where P_a is the competitive price before discrimination

Q_a is the given output

and n_a is the price elasticity of demand (absolute value) in the aggregate market at (P_a, Q_a)

Therefore, using these transformations, the optimal price-discriminating share of output in each market segment, s_{id} , and the corresponding optimal price, P_{id} , can be expressed as follows.

$$(4.18) \quad s_{id} = \frac{1}{2} s_i \left[\frac{n_i}{n_a} + 1 \right]$$

where s_i is the share of given output in the i th

market segment at the competitive price, P_a ,
before discrimination

and other variables are as defined for (4.17)

and

$$(4.19) \quad P_{id} = P_a + \frac{1}{2} P_a \left[\frac{1}{n_i} - \frac{1}{n_a} \right]$$

where variables are as defined for (4.17)

Therefore, for a given output, Q_a , a corresponding competitive price, P_a , the share of output allocated to individual market segments before discrimination, s_i , and the price elasticities of demand at the competitive pricing solution, n_i and n_a , it is possible to obtain a direct estimate of optimal discriminatory prices, P_{id} , and optimal market segment shares, s_{id} , for any number of market segments. This generality of the model allows the economics of price discrimination across a large number of market segments to be investigated with computational ease. The analytical nature of the model also avoids the iterative techniques associated with satisfying the equality of marginal revenue condition, which are normally used in alternative simulation procedures (Edwards, 1970), and which can impose quite a computational burden as the number of markets increase.

4.4.2 Revenue Gains from Price Discrimination

If a marketing institution which allocates output to alternative market segments according to competitive pricing principles subsequently price discriminates

between these market segments, it is, in effect, increasing the average revenue per unit of output sold. In essence, the demand curve has been moved to the right, an effect which has been previously noted (Reeves and Longmire, 1982; Sieper, 1982; Gardner, 1983).

This effect can be incorporated into the model as follows. From equation (4.2), the average revenue for a particular level of output, Q_a , under competitive pricing, is given by P_a . However, after price discrimination, average revenue, P_{ad} , for the same level of output, Q_a , is given by

$$(4.20a) \quad P_{ad} = \frac{\sum_{i=1}^m P_{id} Q_{id}}{\sum_{i=1}^m Q_{id}}$$

where P_{id} is the optimal price after price discrimination in market segment i

and Q_{id} is the optimal output allocated to market segment i after price discrimination

Alternatively,

$$(4.20b) \quad P_{ad} = \sum_{i=1}^m P_{id} s_{id}$$

where s_{id} is the optimal proportion of output allocated to market segment i after price discrimination

and other variables are as previously defined.

Now, for each level of output, Q_a , a level of average revenue after price discrimination, P_{ad} , can be identified, and this relationship can be mathematically

defined as follows. By substituting equations (4.15) and (4.16) into (4.20a), and manipulating, equation (4.21) emerges.

$$(4.21) \quad P_{a,i} = a_i - b_i Q_i + \frac{k}{Q_i}$$

$$\text{where } k = \sum_{i=1}^m \left[\frac{(a_1 - a_m)^2}{4b_i} \right] > 0$$

Now after price discrimination, the number of market segments which remain functional may differ from that under competitive pricing. Recall from Section 4.2 that possible market segments may be ranked such that $a_1 > a_2 > \dots > a_i > \dots > a_m$. If $a_i > MR_a$ after discriminatory pricing, then i market segments will remain functional. Since the corresponding condition under competitive pricing was $a_i > P_0$ for i markets to remain functional, it follows that, in some cases, the number of functional market segments under price discrimination will be greater than under competitive pricing. That is, more elastic market segments, which could be ignored under competitive pricing, may be exploited under discriminatory pricing. However, rather than introduce a further complexity into the model, it will be assumed that, both before and after price discrimination, there are m functional markets.

The discriminatory pricing model developed above will now be graphically illustrated in Figure 4.1. Recall

$$(4.2) \quad P_a = a_a - b_a Q_a$$

This equation is represented by D, and equation (4.21) by

D'. Let Q_a represent a fixed level of output which has been produced. A non-discriminating marketing institution will be able to sell this output at the corresponding competitive price, P_a , thereby receiving total revenue of $P_a Q_a$. On the other hand, a price-discriminating marketing institution will be able to receive a weighted aggregate price, $P_{a,d}$, for the same aggregate output, Q_a , which will return a total revenue of $P_{a,d} Q_a$.

Therefore, for a given output, Q_a , a marketing institution can increase total revenue from price discrimination between market segments by the area $(P_{a,d} - P_a) Q_a$ shown in Figure 4.1.

This graphical illustration can be related to the algebraic model developed. $(P_{a,d} - P_a)$ represents the difference between equations (4.21) and (4.2), which is $\frac{k}{Q_a}$. This represents the revenue gain from price discrimination per unit of output. Therefore, the total revenue increase from price discrimination, $(P_{a,d} - P_a) Q_a$, is equivalent to $\frac{k}{Q_a} Q_a = k$. Hence, total revenue gains from price discrimination, k , are constant, and are not dependent on the level of output for this algebraic model with its linear demand characteristics.

Recall from the previous Chapter that under similar demand conditions, both Reeves and Longmire (1982) and Sieper (1982) graphically illustrated a rightward shift in demand resulting from policies with discriminatory pricing elements. Both studies show an average revenue-quantity relationship after price discrimination which lies to the

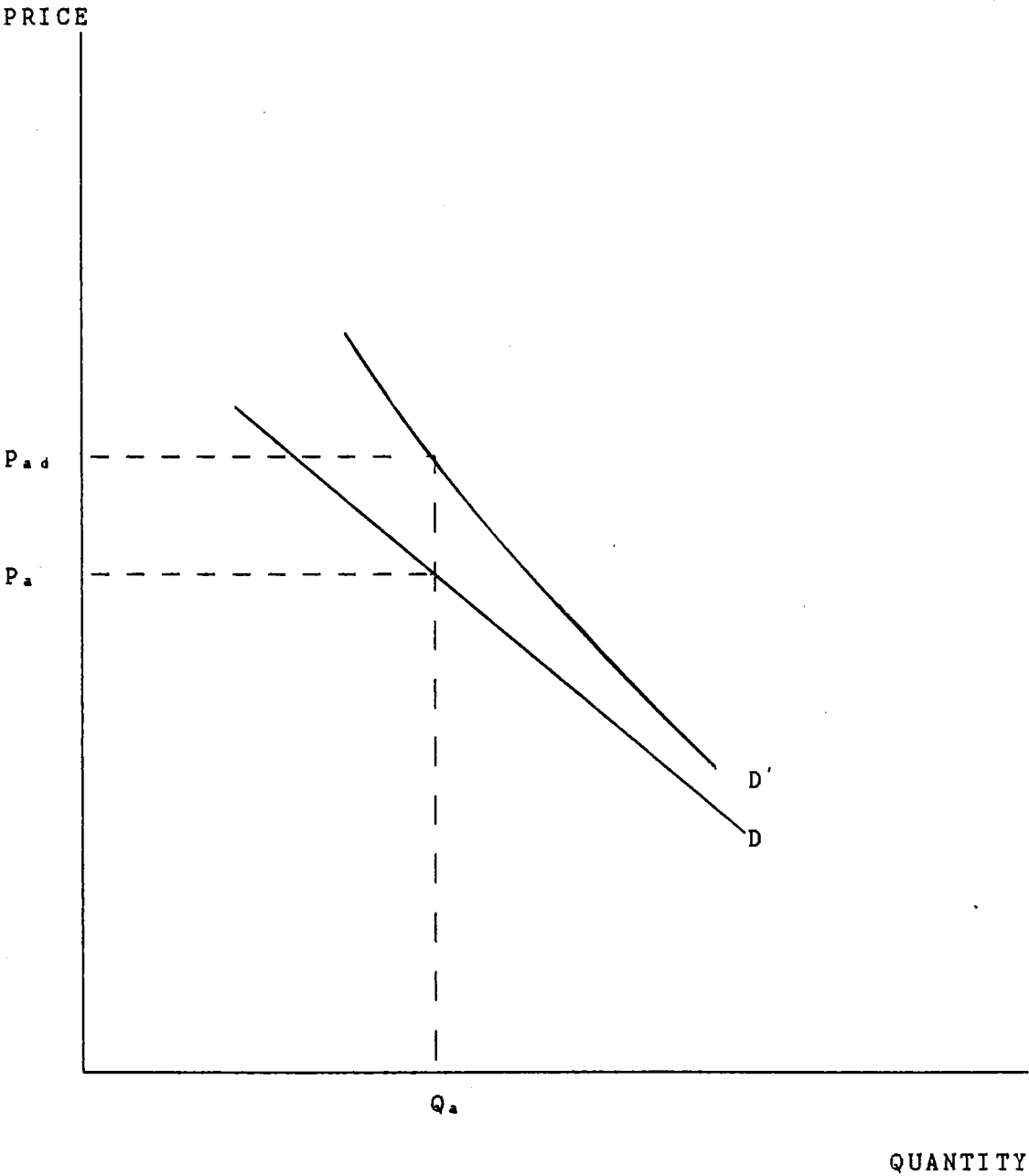


FIGURE 4.1: REVENUE GAINS FROM PRICE DISCRIMINATION

right of an original linear demand curve, and which approaches this curve asymptotically as output increases. As illustrated in Figure 4.1, the effective aggregate 'demand' curve after price discrimination in this model has these properties, and the mathematical relationship between the two demand curves is described precisely by equations (4.2) and (4.21).

The total revenue gains, k , identified above, can be transformed to incorporate elasticities by utilizing equations (4.17a) to (4.17d). The expression for k then becomes

$$(4.22) \quad k = \frac{1}{2} P_a Q_a \sum_{i=1}^m \left[n_i s_i \left[\frac{1}{n_i} - \frac{1}{n_a} \right]^2 \right]$$

where Q_a is the given output

P_a is the competitive price for Q_a

s_i is the proportion of output, Q_a , allocated to market segment i under competitive pricing

n_i and n_a are the price elasticities of demand (absolute value) in market segment i and the aggregate market, respectively, at the competitive price, P_a

The implication of different market characteristics for the magnitude of total revenue gains will be discussed in Chapter 6.

To conclude this Section, the marginal revenue characteristics of the model will be examined. As noted above, total revenue gains from price discrimination are constant with respect to the level of output. This

implies that, for a given level of output, marginal revenue does not change after product has been reallocated among market segments in a discriminatory manner. This can be verified mathematically. As noted previously, if equation (4.2) is multiplied by output, Q_a , to give total revenue, and is then subsequently differentiated with respect to Q_a to give marginal revenue, then equation (4.13b) emerges.

$$(4.13b) \quad MR_a = a_a - 2b_a Q_a$$

where variables are as previously defined.

However, if a similar procedure is followed for the price-discriminating average revenue relationship (4.21), then the same marginal revenue curve emerges. The equality of these two aggregate marginal revenue curves arises from the linear demand characteristics of the model, and will not necessarily be preserved when demand curves in market segments are non-linear (Schmalensee, 1981; Robinson, 1933).

This Section has identified an effective demand relationship for discriminatory pricing across any number of markets which determines the average revenue returned from such activity for any aggregate level of output. This allows the total revenue gains from price discrimination to be investigated, and these gains to be expressed in terms of a given output, the competitive price for that output, the proportion of output allocated to each market segment under competitive pricing, and the price elasticities of demand in each market segment and in the aggregate market under competitive pricing. The next

Section considers the supply response of producers to this increased average revenue which results from price discrimination.

4.5 AN OWN SUPPLY RESPONSE TO DISCRIMINATORY PRICING

A marketing institution which price discriminates between market segments would normally return the increased revenue to producers in the form of a pooled price, and has been defined to do so in the context of this study. Since this pooled price will be higher than the corresponding competitive (non-discriminating) price, it will induce a supply response from producers. This is graphically illustrated in Figure 4.2.

As in Figure 4.1, equation (4.2), the non-discriminating demand curve, is represented by D , and equation (4.21), the discriminating average revenue curve, is represented by D' . Recall from the competitive model developed in Section 4.2, that the supply curve was given by equation (4.3) as follows.

$$(4.3) \quad P_s = c + dQ_s; \quad d > 0, \quad c < a.$$

where variables are as previously defined.

This is represented by S in Figure 4.2.

Competitive equilibrium is given by (P_0, Q_0) . This solution was given by equations (4.5a) and (4.5b).

Now, if a marketing institution allocates the competitive industry output, Q_0 , according to discriminatory pricing principles, it will be able to return suppliers a pooled price, P_{40} . This increased producer price will induce an increased output from

suppliers. After adjustments resulting from this have finally been worked through, the price-discriminating equilibrium will settle at (P_{d1}, Q_1) shown in Figure 4.2.

From this, it becomes obvious that the equilibrium price, P_{d1} , will be greater than the original competitive price, P_0 , but less than the initial discriminatory price, P_{d0} . It is of interest to note that, without price discrimination, the new equilibrium output level of Q_1 would have returned producers a price of P_1 as opposed to P_{d1} .

The change in equilibrium industry output induced by discriminatory pricing, or the supply response, is measured by $(Q_1 - Q_0)$ in Figure 4.2.

These graphical observations can be incorporated into the algebraic model. Equilibrium is ensured by equating the discriminating average revenue, equation (4.21), with supply, equation (4.3). This gives the pooled price, P_{d1} , where

$$(4.23a) \quad P_{d1} = a_s - b_s \left[\frac{a_s - c + X^{0.5}}{2(b_s + d)} \right] + \frac{2k(b_s + d)}{a_s - c + X^{0.5}}$$

and the price discriminating output, Q_1 , where

$$(4.24a) \quad Q_1 = \frac{a_s - c + X^{0.5}}{2(b_s + d)}$$

where $X = (a_s - c)^2 + 4k(b_s + d)$ in both cases

These variables P_{d1} and Q_1 can be transformed into the expressions which incorporate the competitive equilibrium price and output, P_0 and Q_0 , and the price elasticities of

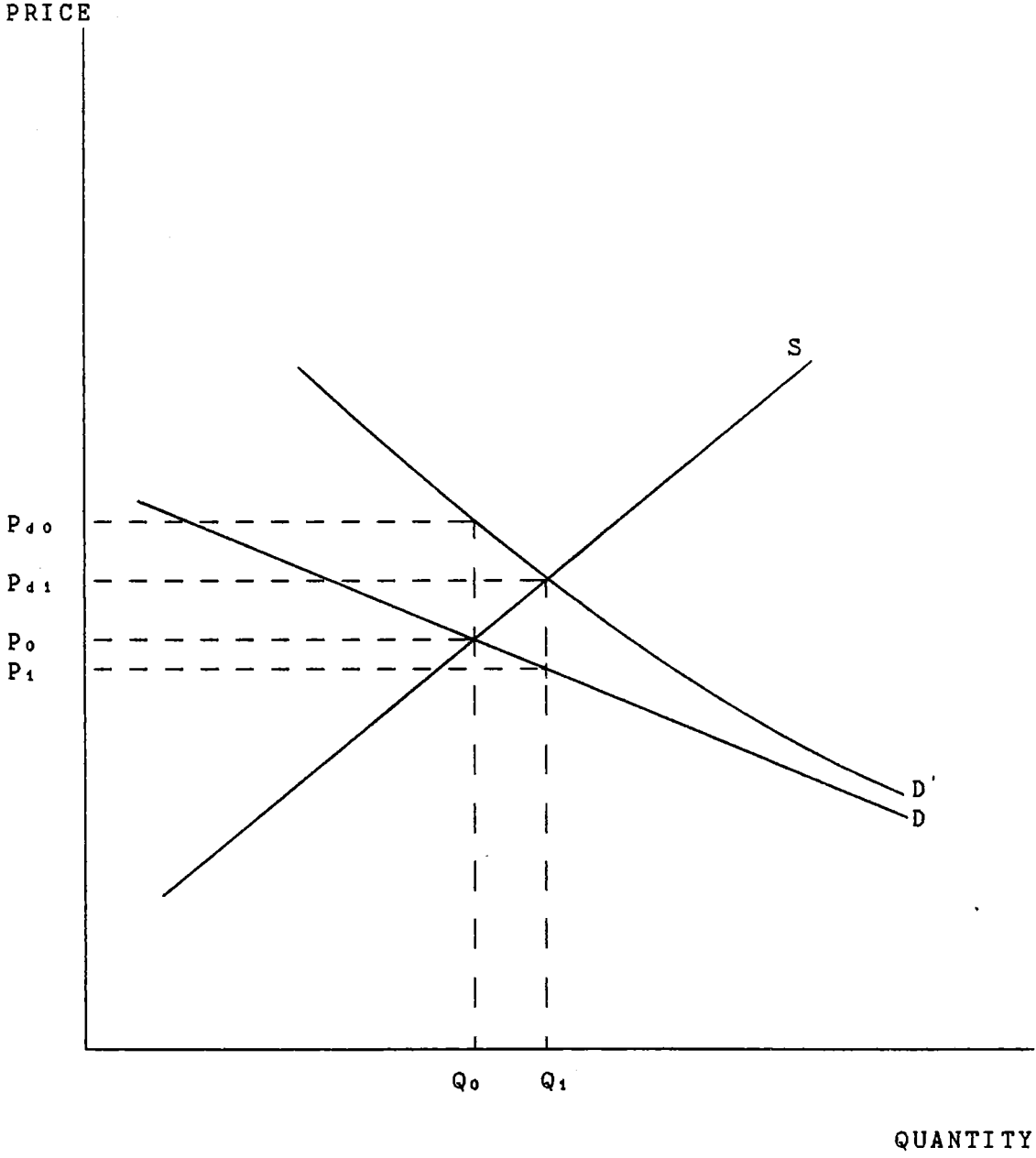


FIGURE 4.2: OWN SUPPLY RESPONSE TO DISCRIMINATORY PRICING

demand and supply at this equilibrium, n_a and e respectively. Before doing this, however, the intercept and slope of the supply curve, equation (4.3) must be transformed in a manner similar to that in equations (4.17a) to (4.17d). This gives

$$(4.17e) \quad c = P_0 \left[1 - \frac{1}{e} \right]$$

and

$$(4.17f) \quad d = \frac{P_0}{Q_0} \cdot \frac{1}{e}$$

where variables are as defined above

This gives

$$(4.23b) \quad P_{d1} = P_0 \left[1 + \frac{1}{2n_a} \right] - \frac{P_0 Z^{0.5}}{Q_0 n_a} + \frac{k}{\frac{1}{2}Q_0 + Z^{0.5}}$$

and

$$(4.24b) \quad Q_1 = \frac{Q_0}{2} + Z^{0.5}$$

$$\text{where } Z = \frac{Q_0^2}{4} + \frac{kQ_0}{P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right]} \quad \text{in both cases}$$

and other variables are as defined above.

The supply response, $Q_1 - Q_0$, or ΔQ , is calculated by subtracting equation (4.5b), which gives Q_0 , from equation (4.24), which gives Q_1 .

$$(4.25a) \quad \Delta Q = \frac{-(a_a - c) + X^{0.5}}{2(b_a + d)}$$

$$\text{where } X = (a_a - c)^2 + 4k(b_a + d)$$

Once again, this can be transformed to competitive equilibrium values and elasticities to give

$$(4.25b) \quad \Delta Q = -\frac{Q_0}{2} + \left[\frac{Q_0^2}{4} + \frac{kQ_0}{P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right]} \right]^{0.5}$$

where values are as defined previously

The implication of this own supply response for producer returns will be investigated in Section 4.7. Meanwhile, the next Section investigates the implications of a competitive supply response in individual market segments for the above model.

4.6 A COMPETITIVE SUPPLY RESPONSE TO DISCRIMINATORY PRICING

As noted in the literature, a competitive supply response will be induced by unilateral discriminatory pricing by a marketing institution which operates in market segments where it does not have a complete monopoly (Rae, 1978; Banks and Mauldon, 1966; Edwards, 1970; Weisenborn, 1969; Freebairn and Gruen, 1977).

Edwards (1970) and Veeman (1972) attempted to take account of this market characteristic by discounting market price elasticities of demand by market share. This yields a price elasticity of demand facing a marketing institution in a market segment which is more elastic than the market price elasticity in that segment. Although such a formulation acknowledges the presence of competitors in these market segments, it does not incorporate their responses to discriminatory pricing (Edwards, 1970).

However, by adapting theoretical work on export demand elasticities (Tweeten, 1967; Cronin, 1979; Bredahl et al, 1979; Tweeten, 1977), such responses can be incorporated into the model. The method reported here is adapted from that first developed by Tweeten (1967) and reported and utilized by Cronin (1979). It derives a formula which estimates the price elasticity of demand facing a marketing institution for its product in a particular (in this case, export) market segment.

Begin with the identity

$$(4.26) \quad X_{i,a} \equiv C_i - (S_i + \sum_{j=1}^n X_{i,j}) \quad i, j \neq a$$

where $X_{i,a}$ is the quantity of a product exported from country a to country i

C_i is consumption of the product in country i

S_i is the quantity of the product supplied by country i

and $\sum_{j=1}^n X_{i,j}$ are the quantities of product imported into country i from all other exporting countries, excluding country a

Now, differentiate equation (4.26) with respect to country a's export price to country i, $P_{i,a}$.

$$(4.27) \quad \frac{\delta X_{i,a}}{\delta P_{i,a}} = \frac{\delta C_i}{\delta P_{i,a}} - \left[\frac{\delta S_i}{\delta P_{i,a}} + \sum_{j=1}^n \frac{\delta X_{i,j}}{\delta P_{i,a}} \right]$$

The export demand elasticity facing country a for the product in question in country i is defined by

$$(4.28a) \quad n_{x_{i,a}} = \frac{\delta X_{i,a}}{\delta P_{i,a}} \cdot \frac{P_{i,a}}{X_{i,a}}$$

By converting equation (4.27) to this elasticity form and re-arranging, it is possible to derive

$$(4.28b) \quad n_{x_{ia}} = \left[\frac{\delta C_i}{\delta P_{ic}} \cdot \frac{P_{ic}}{C_i} \cdot \frac{\delta P_{ic}}{\delta P_{ia}} \cdot \frac{P_{ia}}{P_{ic}} \cdot \frac{C_i}{X_{ia}} \right] \\ - \left[\frac{\delta S_i}{\delta P_{ip}} \cdot \frac{P_{ip}}{S_i} \cdot \frac{\delta P_{ip}}{\delta P_{ia}} \cdot \frac{P_{ia}}{P_{ip}} \cdot \frac{S_i}{X_{ia}} \right] \\ + \sum_{j=1}^n \left[\frac{\delta X_{ij}}{\delta P_{jp}} \cdot \frac{P_{jp}}{X_{ij}} \cdot \frac{\delta P_{jp}}{\delta P_{ia}} \cdot \frac{P_{ia}}{P_{jp}} \cdot \frac{X_{ij}}{X_{ia}} \right]$$

where p_{ic} is the consumer price for the product in country i

p_{ip} is the producer price for the product in country i

p_{jp} is the producer price for the product for export in country j.

Therefore,

$$(4.28c) \quad n_{x_{ia}} = n_i \phi_{ia} \frac{C_i}{X_{ia}} - \left[e_i \theta_{ia} \frac{S_i}{X_{ia}} + \sum_{j=1}^n e_{ij} \theta_{ja} \frac{X_{ij}}{X_{ia}} \right]$$

where n_i is the own price elasticity of demand for the product in country i

e_i is the own price elasticity of supply for the product in country i

e_{ij} is the elasticity of exports from country j to country i with respect to changes in the export price of the product to country i from country a

$\phi_{i,a}$ is the elasticity of consumer price in country i with respect to the export price of the product to country i from country a

$\theta_{i,a}$ is the elasticity of producer price in country i with respect to the export price of the product to country i from country a

$\theta_{j,a}$ is the elasticity of producer price for export product in country j with respect to the export price of the product to country i from country a

$\frac{X_{i,a}}{C_i}$ is the share of the market for the product in country i held by country a

$\frac{X_{i,a}}{S_i}$ is the ratio of quantity supplied by country a to country i to the quantity supplied to country i by its own suppliers

and $\frac{X_{i,a}}{X_{i,j}}$ is the ratio of quantity supplied by country a to country i to the quantity supplied to country i by all other exporters

An alternative method of formulating this export demand elasticity, $n_{x i,a}$, based on excess demand relationships, has been derived by Tweeten (1977) and Bredahl et al (1979). However, if the own price elasticity of excess demand (imports) in country i is not directly observable and has to be estimated from the underlying own price elasticities of demand and supply, then the calculation of $n_{x i,a}$ becomes more complex than

that reported in equation (4.28c). Consequently, this alternative method is not reported.

The above example has been formulated in terms of a marketing institution in one country exporting its product to a number of other countries. However, it is obvious that equation (4.28c) need not apply only to spatial export market segments, but is quite general, and can refer to market segments created in any dimension.

From equation (4.28c), it can be seen that a wide range of factors influence the price elasticity of demand for the product of a marketing institution which operates in a number of market segments. These include market shares, a number of own-price elasticities of demand, own and cross-price elasticities of supply, and price transmission elasticities. As noted in the previous Chapter, some of these factors have been mentioned by authors concerned with a competitive supply response. For example, Abel (1966) noted that the price responsiveness of an import demand relationship will depend on the price responsiveness of both demand and supply in importing countries.

It was noted above that Veeman (1972) and Edwards (1970) attempted to account for the presence of competitors by discounting the market elasticity of demand in a market segment by the market share in that segment. That is,

$$(4.28d) \quad n_{x1a} = n_1 \frac{C_1}{X_{1a}}$$

where $n_{i,a}$ is the export demand elasticity facing country a for a particular product in market segment i

n_i is the price elasticity of demand for the product in market segment i

C_i is consumption of the product in market segment i

and $X_{i,a}$ is exports of the product from country a to market segment i

It is obvious that equation (4.28d) is a truncated version of equation (4.28c) where specialized assumptions have been made. These assumptions, noted by Cronin (1979) are that there is no supply response from competing countries, product from any one source is a perfect substitute for product from any other source, and no trade barriers exist, thereby allowing the product to be cleared on a free international market. Therefore, Veeman (1972) and Edwards (1970) have made some quite restrictive assumptions, although they acknowledge the first two of these.

In summary, a competitive supply response to discriminatory pricing in individual market segments can be incorporated into the price discrimination model, developed in Section 4.5, through its effect on the price elasticity of demand, n_i , facing a marketing institution for its product in each market segment. The next Section of this Chapter returns to this price discrimination model, and manipulates it in order to calculate returns to producers from price discriminating activity.

4.7 RETURNS TO PRODUCERS FROM PRICE DISCRIMINATION WITH SUPPLY RESPONSE

4.7.1 Optimal Quantities and Prices in Individual Market Segments with Supply Response

At this point, a review of the procedures so far modelled is in order. The algebraic model of price discrimination developed has reallocated a given output among market segments according to discriminatory pricing procedures. This results in a higher price being returned to producers, who respond accordingly by increasing output. In addition, competitors in individual market segments respond to this altered pricing behaviour, thereby influencing the price elasticity of demand facing a institution in any particular market segment. Before the returns to producers which result from this procedure are calculated, consideration will be given to the optimal prices and outputs which result from reallocating the aggregate price-discriminating output.

Initially, the given output to be reallocated before any supply response occurs is the output, Q_0 , which resulted under competitive pricing. However, after a supply response to discriminatory pricing has occurred, the aggregate output which results is Q_1 . This was calculated in equation (4.24b). Associated with this increased output, Q_1 , are a set of optimal prices and outputs in individual market segments, which differ from those calculated for the competitive output, Q_0 . These prices and outputs will now be calculated.

The equilibrium price-discriminating output, Q_1 , is optimally allocated among different market segments by applying equation (4.15) to this equilibrium level of output, Q_1 . That is,

$$(4.29) \quad Q_{1id} = \frac{2b_a Q_1 + a_i - a_a}{2b_i}$$

where Q_{1id} is the price-discriminating equilibrium level of output allocated to market segment i

This equation can be transformed to incorporate elasticities and converted to the optimal proportion of output allocated to individual market segments, s_{1id} , all measured at competitive equilibrium, as follows.

$$(4.30) \quad s_{1id} = \frac{n_i}{n_a} s_i + \frac{1}{2} \frac{Q_1}{Q_1} \left[1 - \frac{n_i}{n_a} \right]$$

where Q_1 is the price-discriminating equilibrium output

Q_1 is the level of output allocated to market segment i under competitive equilibrium

s_i is the proportion of competitive output, Q_0 , allocated to market segment i under competitive equilibrium

n_i is the price elasticity of demand (absolute value) in market segment i at competitive equilibrium

n_a is the price elasticity of demand (absolute value) in the aggregate market at competitive equilibrium

Corresponding optimal prices, P_{i1d} , in individual market segments under price-discriminating equilibrium can also be calculated in this manner. Take equation (4.16) for the equilibrium level of output, Q_1 . This gives

$$(4.31a) \quad P_{i1d} = \frac{1}{2} (a_1 + a_2) - b_1 Q_1$$

where variables are as previously defined

Once again this equation can be transformed to incorporate elasticities as follows.

$$(4.31b) \quad P_{i1d} = P_0 \left[1 + \frac{1}{2n_1} + \frac{1}{n_2} \left[\frac{1}{2} - \frac{Q_1}{Q_0} \right] \right]$$

where P_0 and Q_0 are the competitive equilibrium price and output respectively

and other variables are as previously defined

Therefore, optimal prices and outputs in individual market segments can be calculated directly from competitive equilibrium data for the output which results after price discrimination by the use of the above formulae. The discussion will now turn to the total revenue gains which result from this allocation of output.

4.7.2 Revenue Gains from Price Discrimination with Supply Response

An appropriate measure of returns to producers is producer surplus. As noted previously, there are two components of producer surplus, the total revenue received from the sale of a particular level of output, and the opportunity cost of resources employed in producing this

output. This sub-Section considers the total revenue component of producer surplus, and determines how it changes when a marketing institution moves from competitive to discriminatory pricing.

Such revenue gains will also be of interest to government, which allocates the rights to marketing institutions to control the flow of output to alternative market segments. When considering the allocation of such rights, therefore, government will be interested in whether such a policy increases export returns, as well as being concerned with producer returns. Therefore, total revenue, as a measure of export returns, is of interest in its own right.

The total revenue gains from discriminatory pricing relative to competitive pricing are illustrated in Figure 4.3. Competitive equilibrium is given by (P_0, Q_0) , and consequently, total revenue from this pricing strategy is given by $P_0 Q_0$. Similarly, the price-discriminating equilibrium is given by (P_{d1}, Q_1) and total revenue from this strategy by $P_{d1} Q_1$. Therefore, the change in total revenue, ΔTR , which results from discriminatory pricing, is given by

$$(4.32a) \quad \Delta TR = P_{d1} Q_1 - P_0 Q_0$$

where variables are as defined above

This increase in total revenue, ΔTR , can be broken into two components. The first of these is the price discrimination component, or the static increase in total revenue which would result if a given level of output is allocated according to discriminatory rather than

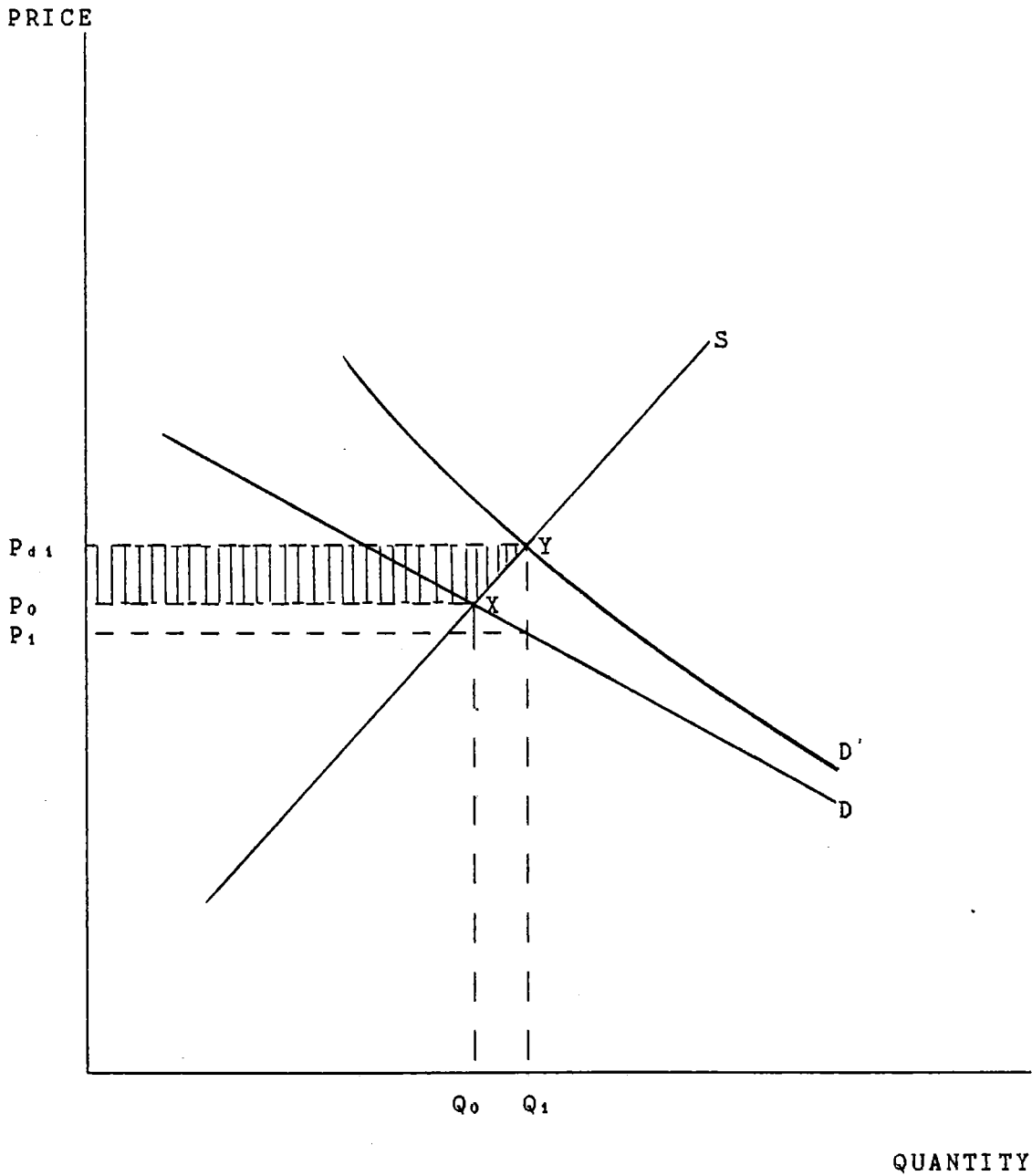


FIGURE 4.3: PRODUCER RETURNS FROM DISCRIMINATORY PRICING

competitive pricing. For the equilibrium price-discriminating output, Q_1 , this is given in Figure 4.3 by $(P_{d1} - P_1)Q_1$.

It was established in equation (4.21) (Section 4.4.2), that this price discrimination component of the increase in total revenue was positive, and constant with respect to output. This price discrimination effect was calculated in equations (4.21) and (4.22) and denoted by k .

The second component of the total revenue change is the supply adjustment resulting from the changed pricing policy. This is equivalent to the revenue change associated with a movement down the non-discriminating demand curve from Q_0 to Q_1 , and is shown in Figure 4.3 by $(P_1Q_1 - P_0Q_0)$.

This supply adjustment may be either positive or negative, depending on the price elasticity of the non-discriminating demand curve in the neighbourhood of the competitive equilibrium solution. When such demand is price inelastic at the competitive equilibrium, the supply adjustment will be negative, whereas it will be positive when demand is price elastic at the discriminating equilibrium.

Therefore, by taking both of these components into account, the change in total revenue, ΔTR , can be expressed as

$$(4.32b) \quad \Delta TR = k + (P_1Q_1 - P_0Q_0)$$

where variables are as previously defined

The magnitude of these total revenue changes can be quantified by utilizing the competitive model developed in Section 4.2, and the price discrimination model developed in Section 4.4. Recall from these models that

$$(4.5a) \quad Q_0 = \frac{a_a - c}{b_a + d}$$

$$(4.5b) \quad P_0 = a_a - b_a \left[\frac{a_a + c}{b_a + d} \right]$$

and

$$(4.24) \quad Q_1 = \frac{a_a - c + X^{0.5}}{2(b_a + d)}$$

$$\text{where } X = (a_a - c)^2 + 4k(b_a + d)$$

and all variables are as previously defined

Now, by substituting equation (4.24), which gives Q_1 , into equation (4.2), P_1 can be derived. This gives

$$(4.33) \quad P_1 = a_a - b_a \left[\frac{a_a - c + X^{0.5}}{2(b_a + d)} \right]$$

$$\text{where } X = (a_a - c)^2 + 4k(b_a + d)$$

and all other variables are as previously defined

By substituting equations (4.5a), (4.5b), (4.24) and (4.33) into equation (4.32b), the following expression for the change in total revenue, ΔTR , is derived.

(4.32c)

$$\Delta TR = k + \frac{(b_a + d)[a_a X^{0.5} - 2b_a k - a_a(a_a - c)] + b_a(a_a - c)(a_a - c - X^{0.5})}{2(b_a + d)^2}$$

$$\text{where } X = (a_a - c)^2 + 4k(b_a + d)$$

and all other variables are as previously defined

This cumbersome algebraic expression can be re-formulated by using the transformations which incorporate the elasticities (equations (4.17a) to (4.17f)). This results in

$$(4.32d) \quad \Delta TR = k + \left[\frac{P_0^2 Q_0^2}{4} + P_0 Q_0 k \frac{n_a e}{n_a + e} \right]^{0.5} \\ - \frac{P_0 Q_0}{2} - k \frac{e}{n_a + e}$$

where k denotes the total revenue gains from price discrimination for a given level of output P_0 and Q_0 are competitive equilibrium price and output respectively n_a is the price elasticity of demand (absolute value) at (P_0, Q_0)

and e is the price elasticity of supply at (P_0, Q_0)

The price discrimination component of the change in total revenue, ΔTR , is given by k , and the supply adjustment component is given by the remainder of the expression on the right-hand side of equation (4.32d).

The implication of differing values of the price elasticity of demand and supply, n_a and e respectively, for the change in total revenue, ΔTR , will be discussed in Chapter 6. The next sub-Section will consider the producer surplus gains which result from discriminatory pricing relative to competitive pricing.

4.7.3 Producer Surplus Gains from Price Discrimination with Supply Response

Under price discrimination, returns to producers may be deduced by calculating producer surplus, PS_d , at the price-discriminating equilibrium, (P_{d1}, Q_1) . By adapting equation (4.6), this is given by

$$(4.33) \quad PS_d = P_{d1}Q_1 - \int_x^{Q_1} (c + dQ) dQ$$

where $x = 0$ if $c \geq 0$

$$x = -\frac{c}{d} \quad \text{if } c < 0$$

and other variables are as previously defined

The change in producer surplus, ΔPS , which results from a move from competitive pricing to discriminatory pricing is calculated by subtracting equation (4.6) from equation (4.33). This gives

$$(4.34a) \quad \Delta PS = (P_{d1}Q_1 - P_0Q_0) - \int_{Q_0}^{Q_1} (c + dQ) dQ$$

where P_{d1} and Q_1 are price-discriminating equilibrium price and output respectively

and P_0 and Q_0 are competitive equilibrium price and output respectively

This change in producer surplus is illustrated in Figure 4.3 by the hatched area P_0XYP_{d1}

There are two components to this increase in producer surplus. The first, $(P_{d1}Q_1 - P_0Q_0)$, is the change in total revenue, ΔTR , which has been investigated in the

previous sub-Section. The second component, $\int_{Q_0}^{Q_1} (c + dQ) dQ$, is the opportunity cost of resources, ΔC , associated with the supply response, ΔQ . By expanding this term, ΔC becomes

$$(4.35a) \quad \Delta C = c(Q_1 - Q_0) + \frac{d}{2}(Q_1^2 - Q_0^2)$$

where variables are as defined above

Solution values for Q_0 and Q_1 were given by equations (4.5b) and (4.24) respectively. If these are substituted into equation (4.35a) which is then rearranged, this gives

$$(4.35b) \quad \Delta C = \frac{2(b_a + d)[-c(a_a - c) + X^{0.5}] - \frac{3d}{2}(a_a - c)^2 \text{ ctd}}{4(b_a + d)^2} + \frac{d(a_a - c)X^{0.5} + \frac{d}{2}X}{2}$$

where $X = (a_a - c)^2 + 4k(b_a + d)$

and other variables are as previously defined

By combining the change in total revenue, ΔTR , (equation (4.32c)), with the opportunity cost of the supply response, ΔC , (equation (4.35b)), and simplifying, the change in producer surplus, ΔPS , becomes

$$(4.34b) \quad \Delta PS = \frac{-d(a_a - c)^2 + d(a - c)X^{0.5} + 2dk(b_a + d)}{4(b_a + d)^2}$$

where $X = (a_a - c)^2 + 4k(b_a + d)$

and other variables are as previously defined

This relationship can also be expressed in terms of elasticities through use of the appropriate equations (4.17a) to (4.17f). This gives

$$(4.34c) \Delta PS = \frac{1}{4e} \left[\frac{2k}{\begin{bmatrix} 1 & 1 \\ - & + \\ n_a & e \end{bmatrix}} - P_0 Q_0 + \left| P_0^2 Q_0^2 + 4k P_0 Q_0 \frac{1}{\begin{bmatrix} 1 & 1 \\ - & + \\ n_a & e \end{bmatrix}} \right|^{0.5} \right]$$

where k denotes the total revenue gains from price discrimination for a given level of output

P_0 and Q_0 are competitive equilibrium price and output respectively

n_a is the price elasticity of demand (absolute value) at (P_0, Q_0)

and e is the price elasticity of supply at (P_0, Q_0)

The implications of differing values of the price elasticity of demand and supply, n_a and e respectively, for the change in producer surplus, ΔPS , which results from a price discrimination policy, will be discussed in Chapter 6.

4.8 CONCLUSION

The objective of this Chapter was to develop a general model of price discrimination which included both an own supply response and a competitive supply response to discriminatory pricing, which would then provide a basis for evaluating price discrimination policies in terms of producer objectives. This was done in a number of steps. A competitive pricing model with multiple market segments was constructed as a benchmark against which to assess discriminatory pricing. The pricing decision which

maximizes producer surplus was then derived. A generalized model of price discrimination which utilized this maximizing condition was constructed, with this model including the required own supply response and competitive supply response. By comparing this discriminatory pricing model with the competitive pricing model previously developed, returns to producers from adopting a discriminatory pricing policy were calculated.

As part of this process, the model calculated a number of important variables. These included

- (a) the total revenue gains, k , which result from reallocating a fixed output among alternative market segments according to discriminatory rather than competitive pricing principles (Equation 4.22, Section 4.4.2),
- (b) the discriminating output level, Q_1 , which results after an own supply response to discriminatory pricing has occurred (Equation 4.24b, Section 4.5),
- (c) the pooled price, P_{d1} , received by producers as a result of discriminatory pricing after an own supply response has occurred (Equation 4.23b, Section 4.5),
- (d) the supply response, or increased output, ΔQ , which occurs as a result of discriminatory pricing (Equation 4.25b, Section 4.5),
- (e) the competitive supply response to discriminatory pricing, which alters the price elasticity of demand facing a marketing institution in a

particular market segment (Equation 4.28c, Section 4.6),

- (f) the optimal proportion of the price discriminating equilibrium output allocated to each market segment, s_{i1} (Equation 4.30, Section 4.7.1),
- (g) the optimal price in each market segment, P_{i1} , given the price-discriminating equilibrium output (Equation 4.31, Section 4.7.1),
- (h) the change in total revenue after discriminatory pricing, ΔTR , which accrues after supply responses have been taken into account (Equation 4.32d, Section 4.7.2),
- (i) the change in producer surplus after discriminatory pricing, ΔPS , which accrues after supply responses have been taken into account (Equation 4.34c, Section 4.7.3).

With the exception of the competitive supply response, each of the above equations was expressed in terms of equilibrium price and output under competitive pricing, P_0 and Q_0 respectively, the share of output allocated to individual market segments under competitive pricing, s_i , the price elasticities of demand (absolute value) in individual market segments and the aggregate market under competitive pricing, n_i and n_a respectively, and the price elasticity of supply under competitive pricing, e . Although such relationships are derived from a linear model, their transformation to these elasticity terms allows some generalizations to be made, since non-linear

relationships can be assumed to be linear over limited ranges.

Equation (4.28c), which includes the competitive supply response expresses the price elasticity of demand facing an institution in a particular market segment in terms of the market demand elasticity in that segment, market shares of alternative suppliers in the segment, and a number of own- and cross-price elasticities of supply and price transmission elasticities.

Therefore, a number of market characteristics influence results from discriminatory pricing. In Chapter 6, the extent to which these market characteristics affect such results will be discussed. This will then make it possible to determine the conditions under which discriminatory pricing, as a major component of a market segmentation policy, is likely to be successful.

CHAPTER 5

ASPECTS OF PROMOTION WITH SUPPLY RESPONSE

The modified third-degree price discrimination model constructed in the previous Chapter included most elements of a marketing mix. However, it did not consider the necessary promotional elements, and the objective of this Chapter is to incorporate promotion into the theoretical model of price discrimination without supply control.

The Chapter is structured in the following way. In Section 5.2, optimal promotion models which are conventionally used in the analysis of agricultural industries will be considered, while in Section 5.3, optimal promotion policies will be investigated under demand characterized by multiple segments. Attention will then be turned to the actual magnitude of returns which producers receive from promotion in different market segments. To this end, the competitive benchmark model established in Chapter 4 will be extended in Section 5.4 to include the influence of demand-shifting promotion. The discriminatory pricing model constructed will be likewise amended at this point. In Section 5.5 the effect on producer returns of product differentiating promotion under discriminatory pricing will be considered. Finally, in Section 5.6, the conclusions of the Chapter will be summarized, and aspects of the effects of promotion to be discussed in Chapter 6 will be identified.

5.2 CONVENTIONAL MODELS OF OPTIMAL PROMOTION USED FOR AGRICULTURAL INDUSTRIES

Few guidelines exist on how to optimally allocate advertising effort among market segments under appropriate agricultural supply-side conditions, and the literature review in Chapter 3 established that this topic has been inadequately investigated. Consequently, in the next two Sections, an attempt will be made to develop decision rules which are appropriate for these circumstances.

One of the earliest models was that of Dorfman and Steiner (1954) and this theorem of optimal advertising has provided the basis for the more complex advertising theorems developed for agricultural marketing (de Boer, 1977; Nerlove and Waugh, 1961; Strak and Gill, 1983).

Dorfman and Steiner (1954) were concerned with how much advertising effort a monopoly firm should direct towards its (aggregate) market. This problem can be mathematically modeled in the following manner. Let a downward sloping demand curve, Q , which shifts out with advertising be represented by

$$(5.1) \quad Q = Q(P, A)$$

where P is price

and A is advertising expenditure.

The profit function, π , is given by

$$(5.2) \quad \pi = PQ(P, A) - C(Q) - A$$

where $C(Q)$ is the total cost of producing output Q

When this profit function, π , is maximized, it yields the optimal advertising decision rule

$$(5.3) \quad \theta = \beta L$$

where θ is the advertising to sales ratio, $A/(PQ)$

β is the advertising elasticity of demand

and L is the Lerner index of monopoly power.

The corresponding product-price decision rule is given by

$$(5.4) \quad L = 1/n$$

where n is the price elasticity of demand

(absolute value)

Obviously, these optimal product-price and advertising decision rules can be expressed in one equation which encapsulates both rules as follows.

$$(5.5) \quad \theta = \beta/n$$

Equation (5.5) has become known as the Dorfman-Steiner theorem, and a convenient summary of the derivation of the theorem in the above format is given by Koutsogiannis (1982).

A monopolist would be optimally advertising when the ratio of advertising to sales is equal to the ratio of the advertising to price elasticities of demand. Obviously, advertising effort would be greater when, at equilibrium, the advertising elasticity is higher or the price elasticity is lower.

Although the above model provides valuable insights into the optimal allocation of advertising effort, its prescriptions are appropriate only for an agency which is facing a single market and has the necessary power to control the level of supply.

De Boer (1977) recognized the first of these limitations, and attempted to extend the Dorfman-Steiner theorem to consider optimal advertising behaviour by a price-discriminating monopolist operating in two market segments. His method for tackling this problem was to utilize optimal discriminatory pricing conditions and to substitute the Dorfman-Steiner theorem of optimal advertising into these.

To elaborate, optimal pricing is practised when marginal revenues in market segments are equal. For the two-segment case, this may be expressed as

$$(5.8) \quad P_1 \left[1 - \frac{1}{n_1} \right] = P_2 \left[1 - \frac{1}{n_2} \right]$$

where P_i is price in market segment i

and n_i is the price elasticity of demand in i .

De Boer (1977) then rearranged the Dorfman-Steiner condition and substituted this into (5.8), yielding

$$(5.9) \quad \frac{\Theta_2 - \beta_2}{\Theta_1 - \beta_1} = \frac{P_1 \beta_2}{P_2 \beta_1}$$

Now, if it is assumed that the optimal price is greater in market segment 1; that is, $P_1 > P_2$, and that the advertising elasticities of demand in both market segments are equal, then according to De Boer (1977) it becomes obvious from equation (5.9) that

$$(5.10a) \quad \Theta_1 < \Theta_2$$

That is, optimal advertising effort should be more intensive in the more price elastic market. (The full mathematical derivation of this and subsequent models is given in Appendix 2.)

However, this result assumes that the advertising elasticities are less than the optimal advertising intensities, which would occur when both demands are price inelastic. If on the other hand, it is assumed that demands are price elastic (which implies that advertising elasticities are greater than optimal advertising intensities), then the opposite result emerges. That is,

(5.10b) $\Theta_1 > \Theta_2$

Furthermore, if the advertising elasticities lie between the optimal advertising intensities, then one of the known positive variables must be negative, which is highly disturbing. In addition, there is no a priori reason to support the conclusion that the relative intensity of advertising effort is dependent on the value of price elasticities of demand.

As a consequence, de Boer's conclusion must be regarded as suspect. It must also be noted that the method of substituting the optimal prescriptions from one model into the optimal prescriptions from another model is a dubious procedure. Therefore, a model of advertising in two market segments was correctly specified, and optimal pricing and promotion decisions derived. The results of this model unambiguously indicated that when advertising elasticities are assumed equal, then

(5.11a) $\Theta_1 > \Theta_2$

In reality, it is probable that a high price elasticity of demand is associated with a high advertising elasticity of demand through cross-elasticity effects (de Boer, 1977; Parish, 1963). To elaborate, a high when high advertising demand elasticity could be expected to occur

is a modification of the Dorfman-Steiner theorem to incorporate agricultural supply-side conditions.

In a simplified version of the Nerlove-Waugh model, demand in the aggregate market, Q , can be represented by

$$(5.12) \quad Q = D(P, A)$$

where P is price

and A is advertising expenditure

and supply is given by

$$(5.13) \quad S = S(P)$$

where S is output supplied at price P .

Producer surplus, PS , can be represented by

$$(5.14a) \quad PS = P \cdot D(P, A) - C(S(P))$$

where $C(S)$, the aggregate cost of production, is the area under the supply curve to the left of S .

Returns to producers, R , are given by

$$(5.14b) \quad R = PS - A$$

or

$$(5.14c) \quad R = P \cdot D(P, A) - C(S(P)) - A$$

where all variables are as previously defined.

In order to derive optimal advertising conditions, this profit function, R , is maximized. However, this must be done subject to the constraint that excess supply is zero. That is,

$$(5.15) \quad D(P, A) = S(P)$$

In order to solve this problem, Nerlove and Waugh (1961) applied the then standard principles of comparative statics to the problem of the incidence of a shift in the demand schedule, which has been documented by Samuelson (1948), rather than the more direct analytical technique

of the Lagrange multiplier which has more recently become the standard methodology for such problems. Proofs of solutions to the simplified version of the Nerlove-Waugh model presented above may be deduced from proofs presented in Appendix 2 for an analagous model presented in Section 5.2.3.

The solution to the Nerlove-Waugh model yields the following advertising decision rule for optimal advertising intensity, θ .

$$(5.16) \quad \theta = \frac{\beta}{|n| + e}$$

where e is the price elasticity of supply

and other variables are as defined for the Dorfman-Steiner model

As with the Dorfman-Steiner theorem, optimal advertising intensity varies directly with the advertising elasticity of demand, and inversely with the absolute value of the price elasticity of demand. However, advertising intensity also varies inversely with the price elasticity of supply. Note that where price response is completely inelastic or the supply response is suppressed, then the Nerlove-Waugh condition collapses to the Dorfman-Steiner theorem.

The Nerlove-Waugh result represents a significant advance over the Dorfman-Steiner theorem and its derivatives, since it incorporates the required supply response by allowing the product-price decision to be determined by market forces. However, for the purposes of the current study, it is deficient in an unmodified form,

since its promotion prescriptions refer only to an aggregate market demand, rather than to promotion in individual market segments. Consequently, in the next Section, the Nerlove-Waugh theorem is extended to multiple market segments.

5.3 EXTENSIONS TO THE NERLOVE-WAUGH THEOREM

5.3.1 Introduction

An obvious extension to the Nerlove-Waugh theorem would be to generalize it out to multiple market segments under competitive pricing assumptions. That is, a single price would prevail in all market segments, and such a price would be market determined by the intersection of the total demand and supply curves. Such a model would provide a competitive benchmark against which optimal advertising expenditure under discriminatory pricing with the appropriate supply conditions could be evaluated. This particular generalization of the Nerlove-Waugh theorem is presented below.

5.3.2 Optimal Advertising in Multiple Market Segments under Competitive Pricing

Assume two independent market segments 1 and 2, which, when combined, make up aggregate industry demand. Individual demand functions in each market segment may then be characterized by

$$(5.17a) \quad Q_1 = D_1(P, A_1)$$

$$(5.17b) \quad Q_2 = D_2(P, A_2)$$

where P is price

Q_i is quantity in market segment i

and A_i is advertising in market segment i .

Aggregate industry demand, Q , may then be represented by

$$(5.17c) \quad Q = Q_1 + Q_2 = D_1(P, A_1) + D_2(P, A_2)$$

The industry supply function can be written as

$$(5.18) \quad Q = S(P)$$

where variables are as previously defined.

Market equilibrium requires that (5.17c) and (5.18) are equal. That is,

$$(5.19) \quad D_1(P, A_1) + D_2(P, A_2) = S(P)$$

Following the methodology of the Nerlove-Waugh theorem outlined above in Section 5.2.1, consider an appropriate measure of producer returns from advertising in each market segment. Let

$$(5.20a) \quad R_1 = PS - A_1$$

$$(5.20b) \quad R_2 = PS - A_2$$

where R_i is returns from advertising in market segment i

A_i is advertising expenditure in market segment i

and PS is producer surplus

Producer surplus, PS , may be represented by

$$(5.21) \quad PS = PQ - \int_0^Q S^{-1}(Q) dQ$$

where P is price

Q is quantity

and $S^{-1}(Q)$ is the inverse of the industry supply curve.

Now consider the effect of a change in advertising in market segment 1 on returns to producers. This is found by differentiating R_1 with respect to A_1 . This gives

$$(5.22a) \quad \frac{\delta R_1}{\delta A_1} = Q \frac{\delta P}{\delta A_1} - 1$$

Similarly, the effect of a change in advertising in market segment 2 on returns to producers is given by

$$(5.22b) \quad \frac{\delta R_2}{\delta A_2} = Q \frac{\delta P}{\delta A_2} - 1$$

It is now required to find expressions for $\frac{\delta P}{\delta A_1}$ and

$\frac{\delta P}{\delta A_2}$. Consider $\frac{\delta P}{\delta A_1}$. Differentiate equations (5.18) and

(5.19) with respect to A_1 . This gives

$$(5.23a) \quad \frac{\delta Q}{\delta A_1} - \frac{\delta D_1}{\delta P} \cdot \frac{\delta P}{\delta A_1} - \frac{\delta D_2}{\delta P} \cdot \frac{\delta P}{\delta A_1} = \frac{\delta D_1}{\delta A_1}$$

$$(5.23b) \quad \frac{\delta Q}{\delta A_1} - \frac{\delta S}{\delta P} \cdot \frac{\delta P}{\delta A_1} = 0$$

Solving the simultaneous equations (5.23a) and (5.23b) for

$\frac{\delta P}{\delta A_1}$ gives

$$(5.24a) \quad \frac{\delta P}{\delta A_1} = \frac{\frac{\delta D_1}{\delta A_1}}{\frac{\delta S}{\delta P} - \left[\frac{\delta D_1}{\delta P} + \frac{\delta D_2}{\delta P} \right]}$$

By a similar procedure, $\frac{\delta P}{\delta A_2}$ can be found to be

$$(5.24b) \quad \frac{\frac{\delta P}{\delta A_2}}{\frac{\delta S}{\delta P} - \left[\frac{\frac{\delta D_1}{\delta P}}{\delta P} + \frac{\frac{\delta D_2}{\delta P}}{\delta P} \right]} = \frac{\frac{\delta D_2}{\delta A_2}}{\frac{\delta S}{\delta P} - \left[\frac{\frac{\delta D_1}{\delta P}}{\delta P} + \frac{\frac{\delta D_2}{\delta P}}{\delta P} \right]}$$

Equations (5.24a) and (5.24b) can then be substituted into equations (5.22a) and (5.22b) to give

$$(5.25a) \quad \frac{\delta R_1}{\delta A_1} = Q \left[\frac{\frac{\frac{\delta D_1}{\delta A_1}}{\frac{\delta S}{\delta P} - \left[\frac{\frac{\delta D_1}{\delta P}}{\delta P} + \frac{\frac{\delta D_2}{\delta P}}{\delta P} \right]}}{\frac{\delta S}{\delta P} - \left[\frac{\frac{\delta D_1}{\delta P}}{\delta P} + \frac{\frac{\delta D_2}{\delta P}}{\delta P} \right]} \right] - 1$$

and

$$(5.25b) \quad \frac{\delta R_2}{\delta A_2} = Q \left[\frac{\frac{\frac{\delta D_2}{\delta A_2}}{\frac{\delta S}{\delta P} - \left[\frac{\frac{\delta D_1}{\delta P}}{\delta P} + \frac{\frac{\delta D_2}{\delta P}}{\delta P} \right]}}{\frac{\delta S}{\delta P} - \left[\frac{\frac{\delta D_1}{\delta P}}{\delta P} + \frac{\frac{\delta D_2}{\delta P}}{\delta P} \right]} \right] - 1$$

Take equation (5.25a). By multiplying the right-hand side by $\left[\frac{P}{-} \right] / \left[\frac{P}{-} \right]$ and manipulating, this can be restated in elasticity terms as

$$(5.26a) \quad \frac{\delta R_1}{\delta A_1} = \frac{\alpha_1}{e - (n_1 s_1 + n_2 s_2)} - 1$$

where α_1 is the marginal gross revenue from market 1 from increased advertising in that market, holding price constant

e is the price elasticity of supply

n_i is the price elasticity of demand in market segment i

and s_i is the share of output, $\frac{Q_i}{Q}$, sold in market segment i .

Similarly,

$$(5.26b) \quad \frac{\delta R_2}{\delta A_2} = \frac{\alpha_2}{e - (n_1 s_1 + n_2 s_2)} - 1$$

where variables are similarly defined.

The condition for first-order maximization of returns to producers requires that equations (5.26a) and (5.26b) be set to zero. That is, when returns are maximized,

$$(5.27a) \quad \frac{\alpha_1}{e - (n_1 s_1 + n_2 s_2)} = 1$$

$$(5.27b) \quad \frac{\alpha_2}{e - (n_1 s_1 + n_2 s_2)} = 1$$

Consider the aggregate advertising intensity, θ . Then

$$(5.28a) \quad \theta = \frac{A}{PQ}$$

where A is total advertising expenditure

P is price

Q is total output

This can be decomposed into

$$(5.28b) \quad \theta = \frac{A_1}{PQ} + \frac{A_2}{PQ} = \theta_1 + \theta_2$$

where A_i is advertising expenditure in market segment i

and θ_i is defined to be the ratio of advertising in market segment i to total sales.

Now, θ is optimal when θ_1 and θ_2 are optimal.

Return to the optimality condition given in equation

(5.27a) above. Now, by definition,

$$(5.29a) \quad \alpha_1 = P \cdot \frac{\delta D_1}{\delta A_1}$$

After manipulation, this becomes

$$(5.29b) \quad \alpha_1 = \frac{PQ}{A_1} \cdot s_1 \cdot \beta_1$$

where β_1 is the advertising elasticity of demand in market segment i

and other variables are as previously defined.

By substituting (5.29b) into (5.27a) and multiplying both sides by $\frac{A_1}{PQ}$, θ_1 may be derived.

$$(5.30a) \quad \theta_1 = \frac{\beta_1 s_1}{e - (n_1 s_1 + n_2 s_2)}$$

Similarly,

$$(5.30b) \quad \theta_2 = \frac{\beta_2 s_2}{e - (n_1 s_1 + n_2 s_2)}$$

where, in both cases, variables are as defined above. Now, equation (5.28b) allows equations (5.30a) and (5.30b) to be aggregated, which gives a measure of the optimal intensity of advertising for the total market, θ , where

$$(5.31a) \quad \theta = \frac{\beta_1 s_1 + \beta_2 s_2}{e - (n_1 s_1 + n_2 s_2)}$$

where variables are as defined above.

This optimal ratio of aggregate advertising to aggregate sales, θ , can be generalized from the two-dimensional market situation considered above to an n -dimensional case. In these circumstances, θ becomes

$$(5.31b) \quad \theta = \frac{\sum_{i=1}^n \beta_i s_i}{e - \sum_{i=1}^n n_i s_i}$$

where variables are as defined above.

This can be further simplified, since

$$(5.32) \quad n_a = \sum_{i=1}^n n_i s_i$$

where n_a is the price elasticity of demand in the aggregate market

and other variables are as previously defined.

Therefore, the optimal aggregate advertising intensity, θ , can be restated as

$$(5.31c) \quad \theta = \frac{\sum_{i=1}^n \beta_i s_i}{e + |n_a|}$$

where variables are as previously defined.

This optimal aggregate intensity, θ , which is applicable to a marketing agency operating in multiple market segments, can be compared with that which emerges under the Nerlove-Waugh theorem where a single aggregate market is characterized. Equation (5.31c) indicates that, as with the Nerlove-Waugh theorem, the optimal aggregate advertising intensity, θ , will be greater when the price elasticity of demand (absolute value) and of supply are relatively low. However, the aggregate advertising demand elasticity of the Nerlove-Waugh theorem has been replaced by advertising demand elasticities in individual market

segments weighted by the proportion of output allocated to that market segment.

Note that equation (5.31c) collapses to the Nerlove-Waugh theorem when individual market segments are not differentiated from each other. In such a case, the advertising elasticities of demand in individual market segments, the β_i 's, could each be viewed as being equal to the aggregate advertising elasticity of demand, β . By substituting this into equation (5.31c), the Nerlove-Waugh condition emerges. That is,

$$(5.31d) \quad \theta = \frac{\beta}{e + |n_a|}$$

where variables are as previously defined.

The analysis to this point has extended the Nerlove-Waugh theorem to multiple market segments where the same price prevails in each segment, and this price is market-determined by the intersection of the supply curve with the aggregate demand curve. The most important result to emerge has been a measure of the optimal aggregate advertising intensity for the multiple market-segment case. The analysis will now be extended to consider the optimal allocation of this advertising expenditure to alternative market segments.

The ratio of optimal advertising in market segment 1 to total aggregate sales, θ_1 , was given in equation (5.30a).

In general,

$$(5.30c) \quad \theta_1 = \frac{\beta_1 s_1}{e + |n_a|}$$

where variables are as previously defined.

Now, the optimal proportion of advertising allocated to market segment i , Γ_i , is given by

$$(5.33a) \quad \Gamma_i = \frac{A_i}{A}$$

where variables are as previously defined.

By dividing equation (5.30c) above by equation (5.31c), this optimal proportion of advertising expenditure, Γ_i , emerges.

$$(5.33b) \quad \Gamma_i = \frac{\beta_i s_i}{\sum_{i=1}^n \beta_i s_i}$$

where variables are as previously defined.

That is, the optimal proportion of advertising allocated to market segment i , Γ_i , is given by the ratio of the advertising elasticity of demand weighted by the share of output allocated to that segment to the sum of all such weighted average advertising elasticities. In essence, the more responsive a market segment is to advertising, or the more important a segment is in terms of output allocated to it, the greater the proportion of total advertising allocated to it.

In the above analysis, the advertising intensity in a particular market segment i , θ_i , has been defined as the ratio of advertising in that market segment, A_i , to total sales, PQ . This advertising intensity can be redefined in terms of sales in market segment i , PQ_i , as follows.

$$(5.34a) \quad \theta_i' = \frac{A_i}{PQ_i}$$

Therefore, by multiplying equation (5.30c) by the

reciprocal of the share of output allocated to a particular market segment, $\frac{1}{s_i}$, the optimal advertising intensity in that market segment, θ_i' , is given by

$$(5.34b) \quad \theta_i' = \frac{\beta_i}{e + |n_s|}$$

where variables are as previously defined.

That is the ratio of optimal advertising to the actual value of sales generated in a particular market segment is directly related to the advertising elasticity of demand in that segment, and inversely related to the aggregate price elasticity of demand (absolute value) and the price elasticity of supply

A further useful extension to the above analysis is to directly compare the optimal allocation of advertising effort to two market segments. In this case,

$$(5.35a) \quad \frac{\theta_1'}{\theta_2'} = \frac{\beta_1}{\beta_2}$$

where variables are as previously defined.

Hence, the ratio of optimal advertising per unit sales in market segment 1 to that in market segment 2 is equivalent to the ratio of the corresponding advertising elasticities of demand.

In summary, the above sub-section has extended the Nerlove-Waugh theorem of optimal advertising from a single market to multiple market segments, where the same price prevails in each market segment. Four basic conditions emerge from this model as follows.

- (a) The optimal aggregate advertising intensity is identical to that under the Nerlove-Waugh

theorem, with the exception that the advertising demand elasticity for the Nerlove-Waugh aggregate market case is replaced by the sum of advertising elasticities in individual market segments weighted by the share of output allocated to these segments.

- (b) The optimal allocation of advertising expenditure to an individual market segment is given by the ratio of the advertising elasticity of demand in that segment weighted by the share of output allocated to that segment to the sum of all such weighted averages.
- (c) The optimal ratio of advertising to sales generated in a particular market segment is identical to that for the Nerlove-Waugh aggregate market theorem with the exception that the Nerlove-Waugh advertising elasticity is replaced by the advertising elasticity in the relevant market segment. Note therefore, that the optimal advertising intensity in any particular market segment is dependent on the aggregate price elasticity of demand and not on the price elasticity of demand in the particular segment in question.
- (d) Finally, the ratio of optimal advertising per unit sales in market segment i to that in market segment j is equivalent to the ratio of the corresponding advertising elasticities of demand.

These results indicate how advertising effort may be directed when the aggregate market is disaggregated into individual segments, and as such, provide a significant advance over the prescriptions of the Nerlove-Waugh model. They have intuitive economic appeal, since they indicate that more advertising effort should be directed towards those market segments where demand is more responsive to advertising, or where a relatively large proportion of output is sold. Since the same price is charged in all market segments (that is, no attempt is made to exploit any monopoly advantage in individual market segments), it is not surprising that the aggregate price elasticity of demand rather than these individual demand elasticities, influences the intensity of advertising effort.

5.3.3 Optimal Advertising under Discriminatory Pricing with a Competitive Supply Response

The model presented above extends the Nerlove-Waugh theorem to multiple market segments, where one price prevails in all segments and is determined by the intersection of the aggregate demand curve with the supply curve. However, where an institution has control over the allocation of output to alternative market segments, it is in a position to optimize the pricing decisions in these segments.

Therefore, a further extension to the Nerlove-Waugh theorem would involve investigating optimal advertising under discriminatory pricing for the situation where producers are free to respond to the pool price which they

receive from this pricing policy. Since it includes the appropriate supply-side conditions, such a model would provide more appropriate prescriptions for optimal advertising than the de Boer (1977) model outlined in Section 5.2.

This discriminatory pricing model will now be outlined. Its prescriptions will then be compared to those emerging from the Nerlove-Waugh extension developed in Section 5.3.1 above, since this provides a competitive pricing benchmark against which optimal advertising under discriminatory pricing can be evaluated.

Consider a market which can be divided into two segments. In such a case, demand can be represented by

$$(5.35) \quad Q = Q_1(P_1, A_1) + Q_2(P_2, A_2)$$

where, as previously,

Q_i is demand in the i th segment

P_i and A_i are price and advertising, respectively, in the i th segment

and Q is total demand.

Supply is given by

$$(5.36) \quad S = S(P^*)$$

where P^* is the return per unit of output, or the pool price, received by the producer. (Note that the pool price was denoted in general terms by P_{ad} in the algebraic model. P^* is identical to P_{ad}).

This pool price, P^* , is calculated as

$$(5.37) \quad P^* = \frac{P_1 Q_1(P_1, A_1) + P_2 Q_2(P_2, A_2)}{Q_1(P_1, A_1) + Q_2(P_2, A_2)}$$

Following the Nerlove-Waugh theorem, aggregate returns to producers are given by

$$(5.38) \quad R = P_1 Q_1(P_1, A_1) + P_2 Q_2(P_2, A_2) - C(S(P^*)) - A_1 - A_2$$

where $C(S)$ is the aggregate cost of production, and is represented by the supply curve to the left of S

and other variables are as previously defined.

As with the extension to the Nerlove-Waugh theorem considered above, a marketing agency is constrained to adopt policies such that it sells all the output supplied at the price producers receive; in this case, the pool price, P^* . That is,

$$(5.39a) \quad Q_1(P_1, A_1) + Q_2(P_2, A_2) = S(P^*)$$

or, alternatively,

$$(5.39b) \quad x(P_1, P_2, A_1, A_2) \equiv S(P^*(P_1, P_2, A_1, A_2)) - Q_1(P_1, A_1) - Q_2(P_2, A_2) = 0$$

That is, excess supply, x , is required to equal zero.

Explicitly, the marketing agency solves

$$(5.40) \quad \max_{P_1, P_2, A_1, A_2} R(P_1, P_2, A_1, A_2) \text{ subject to } x(P_1, P_2, A_1, A_2) = 0$$

The mathematical solution to this problem has been derived by Martin et al (1986). This publication is reproduced as Appendix 4, in order to allow proofs to the equations presented below to be verified.

Let λ be the Lagrange multiplier associated with the

constraint in (5.40). The Lagrange equations can be written in the form

$$(5.41a) \quad \frac{\frac{\delta R}{\delta P_1}}{\frac{\delta x}{\delta P_1}} = \frac{\frac{\delta R}{\delta P_2}}{\frac{\delta x}{\delta P_2}} = \frac{\frac{\delta R}{\delta A_1}}{\frac{\delta x}{\delta A_1}} = \frac{\frac{\delta R}{\delta A_2}}{\frac{\delta x}{\delta A_2}} = \lambda$$

or, alternatively,

$$(5.41b) \quad \frac{Q_1 + \left[P_1 - \frac{\delta C}{\delta S} \right] \frac{\delta Q_1}{\delta P_1}}{\frac{\delta S}{\delta P^*} - \frac{\delta P^*}{\delta P_1} - \frac{\delta Q_1}{\delta P_1}} = \frac{Q_2 + \left[P_2 - \frac{\delta C}{\delta S} \right] \frac{\delta Q_2}{\delta P_2}}{\frac{\delta S}{\delta P^*} - \frac{\delta P^*}{\delta P_2} - \frac{\delta Q_2}{\delta P_2}}$$

$$= \frac{\left[P_1 - \frac{\delta C}{\delta S} \right] \frac{\delta Q_1}{\delta A_1} - 1}{\frac{\delta S}{\delta P^*} - \frac{\delta P^*}{\delta A_1} - \frac{\delta Q_1}{\delta A_1}} = \frac{\left[P_2 - \frac{\delta C}{\delta S} \right] \frac{\delta Q_2}{\delta A_2} - 1}{\frac{\delta S}{\delta P^*} - \frac{\delta P^*}{\delta A_2} - \frac{\delta Q_2}{\delta A_2}}$$

After manipulation, these first-order conditions for the maximization of aggregate producer returns can be expressed in the following form.

$$(5.42) \quad \frac{P_1}{|n_1|} - \frac{m}{Q_1} = \frac{P_2}{|n_2|} + \frac{m}{Q_2}$$

$$= \frac{\frac{A_1}{\beta_1} - m}{Q_1 - \frac{A_1}{\beta_1} \cdot \frac{\epsilon}{P^*}} = \frac{\frac{A_2}{\beta_2} + m}{Q_2 - \frac{A_2}{\beta_2} \cdot \frac{\epsilon}{P^*}}$$

where $m = (P_1 - P_2) Q_1 Q_2 / (Q_1 + Q_2)$

n_i is the price elasticity of demand in market segment i

β_1 is the advertising elasticity of demand in market segment 1

ϵ is the price elasticity of supply

and other variables are as defined above.

Optimal advertising policies given optimal pricing can now be determined by making the appropriate pairwise comparisons between equations in (5.42). By considering the first and third such equations in (5.42), the optimal advertising intensity in market segment 1, θ_1 , can be derived.

$$(5.43a) \quad \theta_1 = \frac{\beta_1}{(|n_1| + \epsilon) - |n_1|(1 + \epsilon)(1 - \frac{P^*}{P_1})}$$

where θ_1 is defined as the ratio of advertising in market segment 1 to producer returns from that segment (A_1/P^*Q_1)

and other variables are as defined above.

Similarly, in the second market segment,

$$(5.43b) \quad \theta_2 = \frac{\beta_2}{(|n_2| + \epsilon) - |n_2|(1 + \epsilon)(1 - \frac{P^*}{P_2})}$$

In general,

$$(5.43c) \quad \theta_i = \frac{\beta_i}{(|n_i| + \epsilon) - |n_i|(1 + \epsilon)(1 - \frac{P^*}{P_i})}$$

where variables are as defined above.

The formula presented in (5.43c) collapses to the

Nerlove-Waugh theorem when a single aggregate market is assumed. That is,

$$P_1 = P^*, \text{ so } |n_1|(1+e)\left(1 - \frac{P^*}{P_1}\right) \text{ disappears, } \beta_1 = \beta, \text{ and}$$

$n_1 = n_a$. That is, (5.43c) is reduced to

$$(5.31d) \quad \theta = \frac{\beta}{e + |n_a|}$$

where variables are as previously defined.

A comparison will firstly be made between the advertising prescriptions which emerge from this model and those of the Nerlove-Waugh model.

Assume that the price elasticity of demand in the first market is less than that in the second market. That is,

$$(5.44) \quad |n_1| < |n_2|$$

Given optimal (discriminatory) pricing, this implies that

$$(5.45) \quad P_1 > P_2$$

or

$$(5.46) \quad P_1 > P^* > P_2$$

Substituting (5.46) into (5.43a) and (5.43b) implies that

$$(5.47a) \quad \theta_1 > \frac{\beta_1}{|n_1| + e}$$

and

$$(5.47b) \quad \theta_2 < \frac{\beta_2}{|n_2| + e}$$

That is, a price-discriminating marketing agency should choose advertising policies such that in the market segment with the lower (higher) price elasticity of

demand, the ratio of advertising to producer payments should exceed (be less than) the ratio of the advertising elasticity in that market to the sum of the demand and supply elasticities in that market. That is, relatively more (less) advertising effort (as measured by the ratio of advertising to producer returns) would be directed to the less (more) price elastic market segment than would be the case if this was the only market faced by the agency (that is, the Nerlove-Waugh situation).

Finally, a comparison will be made with the prescriptions which arise from this discriminatory pricing extension to the Nerlove-Waugh model, and the single (competitive) price extension considered in the previous sub-Section.

Recall (5.47a) and (5.47b) above. Under the convention (5.44) that $|n_1| < |n_2|$, these imply that

$$(5.48) \quad \frac{A_1}{P^* Q_1 \beta_1} > \frac{1}{|n_1| + e} > \frac{1}{|n_2| + e} > \frac{A_2}{P^* Q_2 \beta_2}$$

That is,

$$(5.49a) \quad \frac{A_1}{Q_1 \beta_1} > \frac{A_2}{Q_2 \beta_2}$$

or

$$(5.49b) \quad \frac{A_1 / Q_1}{A_2 / Q_2} > \frac{\beta_1}{\beta_2}$$

That is, the ratio of advertising per unit sales in the less price elastic market to that in the more price elastic market exceeds the ratio of the corresponding advertising elasticities.

Now recall equation (5.35a) from the previous Section,

where pricing across market segments was competitive. In this case,

$$(5.35a) \quad \frac{\theta_1'}{\theta_2'} = \frac{\beta_1}{\beta_2}$$

where variables are as previously defined.

This implies that

$$(5.35b) \quad \frac{A_1/Q_1}{A_2/Q_2} = \frac{\beta_1}{\beta_2}$$

under these pricing assumptions. That is, the ratio of advertising per unit sales in one market segment to that in the other market segment equals the ratio of the corresponding advertising elasticities.

Therefore, under optimal pricing, relatively more (less) advertising effort (as measured by advertising per unit sales) is directed to the less (more) price elastic segment than under uniform pricing across these segments.

At this point, it is appropriate to summarize this sub-section. The Nerlove-Waugh theorem of optimal advertising has been extended from a single market to multiple market segments, where optimal (discriminatory) pricing policies are practised in these segments.

Three conclusions emerge from this model.

- (a) The formula for the optimal aggregate advertising intensity in a particular market segment is similar to that which emerges under the aggregate Nerlove-Waugh theorem. However, the denominator of this expression in the above extension includes a second term which is a function of the

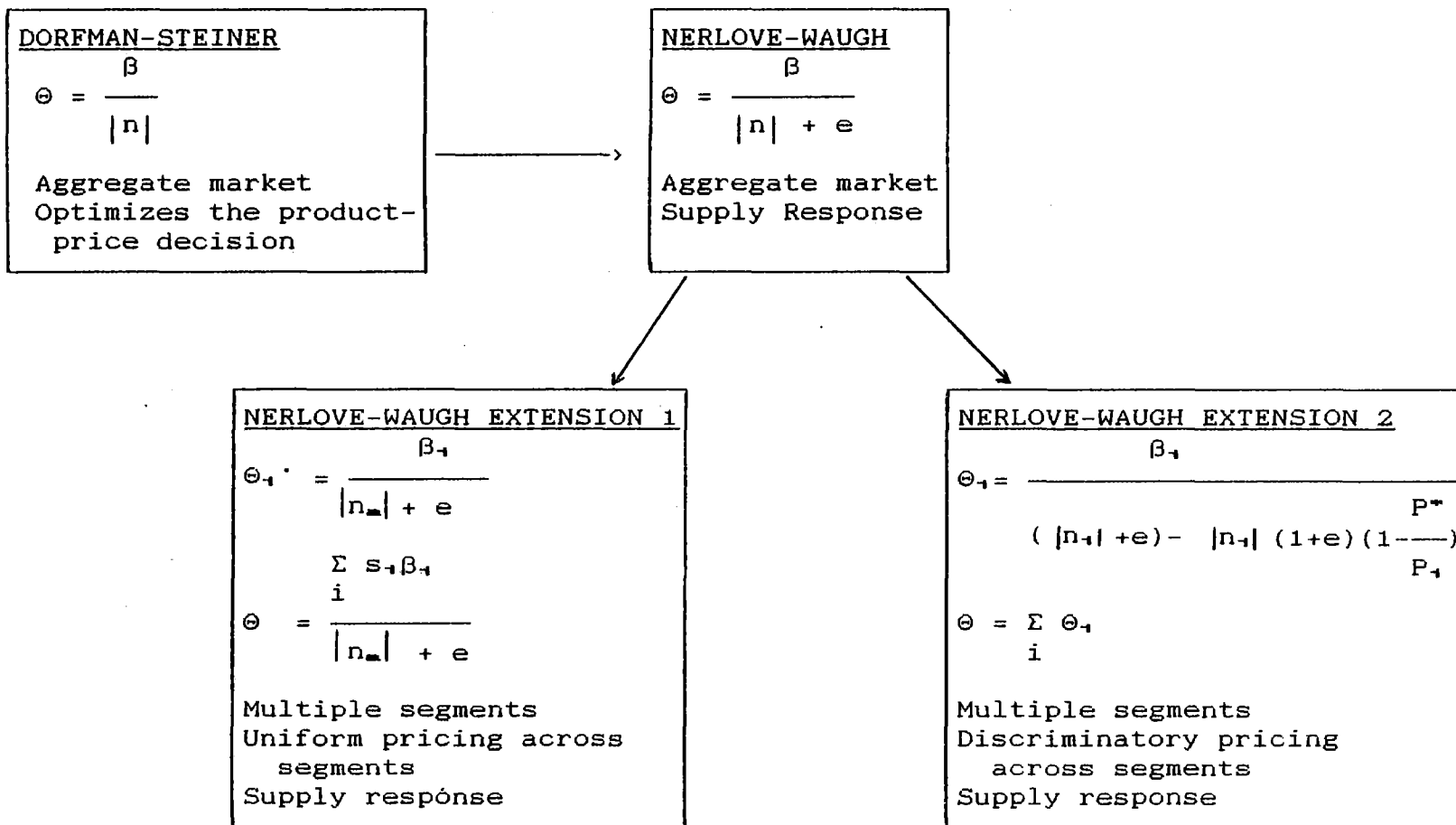
price elasticity of demand in the market segment in question, the price elasticity of supply, and the ratio of the pool price to the optimal price in the market segment.

- (b) For a two-segment case, the optimal advertising intensity in the less price elastic segment is greater than it would be had that segment constituted the aggregate market (the Nerlove-Waugh case). Conversely, for the more price elastic market segment, this intensity would be correspondingly less than it would be under Nerlove-Waugh aggregate market assumptions.
- (c) Assuming two market segments once again, relatively more advertising effort is directed to the less price elastic market segment under discriminatory pricing than would be the case under competitive pricing across these two market segments (the extension to Nerlove-Waugh considered in Section 5.3.2). Conversely, relatively less advertising effort is directed to the more price elastic segment.

5.3.4 Summary of Nerlove-Waugh Extensions

The inclusion of a supply response in models of optimal promotion with multiple market segments does not appear to have been investigated in the agricultural economics literature. Therefore, the objective of this Section was to consider models with this feature.

FIGURE 5.1: MODELS SHOWING OPTIMAL ADVERTISING INTENSITIES UNDER
DIFFERENT MARKET CONDITIONS



The Nerlove-Waugh theorem of optimal advertising was extended to multiple market segments under two sets of pricing assumptions. The first model considered uniform pricing across market segments, while the second incorporated discriminatory pricing between such segments.

The results of these alternative models are summarized in Figure 5.1. This Figure also indicates their relationships to the conventional models employed in the analysis of optimal advertising in agricultural industries, which were reviewed in the previous Section.

The conclusions which emerge from these Nerlove-Waugh extensions would seem to be supported by intuition. That is, more advertising effort is directed towards those market segments where demand is more responsive (as represented by higher advertising elasticities), and where pricing policies are also optimal, more advertising effort is directed to those segments where greater potential exists for appropriating consumer surplus (that is, demand is less price elastic).

These results enhance the general understanding of how advertising effort might be more profitably allocated under market assumptions which are more realistic than those conventionally employed to prescribe the direction of advertising effort in typically structured agricultural industries.

As with the discriminatory pricing model developed in the previous Chapter, it would be desirable to incorporate these optimal promotion formulae into a full marketing mix model where actual producer gains are calculated.

Appropriate market conditions such as price elasticities of demand and supply and advertising elasticities of demand could then be varied to determine the conditions under which the full range of market segmentation policies are likely to yield high returns to producers. The possibility of constructing such a model will now be investigated.

5.4 THE INFLUENCE OF GENERIC PROMOTION ON PRODUCER RETURNS

5.4.1 Introduction

In the previous Section, the optimal ratio of advertising to producer returns in a particular market segment was calculated for discriminatory pricing policies and for the competitive pricing benchmark. These optimal ratios were expressed in terms of appropriate elasticities.

Researchers working with analogous models (Nerlove and Waugh, 1961; Strak, 1983) have attempted to comment on whether producer agencies were over-advertising or under-advertising by using such formulae in the following way. Existing elasticities were used to calculate an 'optimal' advertising to sales ratio. This 'optimal' ratio was then compared to the actual ratio, and on the basis of this comparison, comment was then made on whether advertising effort should be more or less intensive.

While such an approach gives an indication of whether current advertising effort is desirable, it should be used with some caution. This is because the formulae for optimal advertising intensities which are derived from such models are expressed in terms of elasticities which prevail when all variables are in equilibrium. However, the elasticities which are actually used to calculate these ratios are the observed elasticities, which will not necessarily be identical to those prevailing at the equilibrium.

Ideally, the present study requires an estimate of optimal advertising expenditure which could be incorporated into the demand side of the model, which would then be used to calculate producer returns from optimal marketing strategies. However, it would be unwise to attempt to derive optimal promotion expenditure on the basis of the formulae developed in the previous Section because of the above caution. Therefore, the linkage between any given, as opposed to optimal, level of advertising expenditure and increased output which may result from this will be considered.

The linkage between these variables may be envisaged by the following relationship.

$$(5.50a) \quad V_i = \frac{\delta Q_i}{\delta A_i} \cdot \Delta A_i$$

where V_i is the increase in output in market segment i associated with increased advertising expenditure

$\frac{\delta Q_i}{\delta A_i}$ denotes the responsiveness of output to advertising expenditure in segment i

and ΔA_i is the increase in advertising expenditure in segment i .

Equation (5.50a) may be alternatively expressed as

$$(5.50b) \quad V_i = \beta_i \cdot \frac{Q_i}{A_i} \cdot A_i$$

where β_i is the advertising elasticity of demand at (A_i, Q_i) in segment i

A_i is advertising expenditure in segment i

Q_i is output in segment i .

and other variables are as previously defined.

For equation (5.50b) to be used with plausibility, the response of advertising to sales in segment i , $\frac{\delta Q_i}{\delta A_i}$, which is embedded in the advertising elasticity, β_i , must be assumed constant over the range of increased output, V_i . However, empirical evidence suggests that the response of sales to advertising decreases as advertising levels increase (Simon and Arndt, 1980). Such an assumption was implicit in the calculation of optimal advertising intensities undertaken in the previous Sections, since if this were not the case, then producer returns could always have been increased by further advertising. Appropriate restrictions on the behaviour of the sales response to advertising would presumably have been revealed by the second-order conditions, which were not calculated in the previous Sections. This task would have been very complex mathematically, and such an

omission appears to be standard practice when deriving optimal formulae of this type.

In practice, however, very little empirical information exists on these sales responses to advertising in agricultural industries (Strak, 1983) and in fact, not a great deal is known about the advertising elasticities themselves in this sector (Quilkey et al, 1986). Therefore, it may not be possible to incorporate the above quantity change into an appropriate model and to draw meaningful inferences on the influence of alternative advertising elasticities and levels of advertising expenditure on producer returns.

Instead, a more partial approach will be adopted, and the influence on producer returns of a given change in demand which results from promotion expenditure will be investigated. In order to do this, the components of any demand change from advertising will be defined.

Promotion could conceivably influence demand in two ways. The first is by increasing output at any given price, and the second is by altering the price elasticity of demand facing a marketing agency at this given price. In the literature, the first effect is generally termed a generic promotion effect, whereas the second is referred to as a product-differentiating effect. These distinctions will be preserved in this study, and will be considered separately to ensure that results which emerge can be related in a comparable manner to those emerging from other studies which have recognised this distinction between promotion effects.

However, it must be emphasized that when the term 'generic' is used in the context of this study, it refers to the ability of a marketing agency to increase demand at all price levels in a particular market segment. This contrasts to the more conventional definition of the term, which infers that demand for product from all sources, including competitors, is increased by this type of advertising. Schultz and Wittink (1976) and May (1977) discuss a range of intermediate variants of the conventionally defined advertising effects.

In the remainder of this Section, the influence on producer returns of a given generic shift in demand which results from advertising in alternative market segments will be investigated. This will be done under discriminatory pricing, and under benchmark competitive pricing assumptions by incorporating generic promotion into the discriminatory and competitive pricing models developed in the previous Chapter.

5.4.2 A Competitive Pricing Model with Generic Promotion

Consider the competitive pricing model developed in the previous Chapter. Recall that demand in the i th market segment was represented by

$$(4.1b) \quad P_i = a_i - b_i Q_i; \quad a_i, b_i > 0$$

where P_i and Q_i are price and quantity respectively, in market segment i .

Assume that a fixed amount of promotion in market segment i yields a generic effect and increases demand by V_i at all price levels. As noted by Edwards (1984), a

parallel shift in demand such as this implies that each consumer is willing to pay an identical increased price for the product, and this increased price applies to all quantities on each consumer's demand curve.

It has been pointed out by researchers dealing with analogous supply shifts that the effects of such shifts on benefits to producers and consumers may be quite sensitive, in certain circumstances, to the type of shift assumed (Lindner and Jarrett, 1978; Rose, 1980, Edwards and Freebairn, 1984). However, concern is focused in this Section on returns from generic promotion under different pricing regimes. Even if the magnitude of producer returns is influenced by whether a promotion-induced demand shift is convergent, divergent or parallel, it seems reasonable to assume that the relative relationship between returns from competitive or discriminatory pricing will not be altered where an identical type of demand shift is assumed under either pricing regime.

Such an assumption acquires further plausibility when an analogous 'neighbourhood' assumption to that outlined in the previous Chapter is invoked. That is, if equilibrium changes induced by promotion are essentially incremental, the resulting solution under an assumed parallel demand shift will still be a reasonable approximation of the actual solution when the true demand shift is not parallel.

In addition to the above considerations, a parallel demand shift also has computational appeal, since the

algebra involved under alternative demand shifts is quite burdensome, particularly for the discriminatory pricing model.

Hence, demand in the i th market segment after generic promotion may be represented by

$$(5.51a) \quad P_{i,p} = a_i - b_i Q_i + b_i V_i$$

where Q_i is output in market segment i

V_i is the absolute increase in demand after generic promotion in the i th market segment

and $P_{i,p}$ is the price in market segment i after generic promotion.

Therefore, demand in the aggregate market is given by

$$(5.51b) \quad P_{a,p} = a_a - b_a Q_a + b_a V_a$$

where Q_a is output in the aggregate market

$$V_a = \sum_{i=1}^n V_i, \text{ which is the sum of the increase in}$$

demand after generic promotion in each market segment

and $P_{a,p}$ is price in the aggregate market after generic promotion.

That is, generic promotion effects are essentially captured in the intercept term.

From Section (4.2) of the previous Chapter, recall the supply curve.

$$(4.3) \quad P_s = c + dQ; \quad d > 0, \quad c < a_a$$

where P_s and Q are supply price and output respectively.

Therefore, after generic promotion in a market segment or segments, equilibrium is given by

$$(5.52) \quad P_{a,p} = P_s$$

This yields the following equilibrium price, $P_{o,p}$, and output, $Q_{o,p}$.

$$(5.53a) \quad P_{o,p} = a_s - b_s \left[\frac{a_s - c + b_s V_s}{b_s + d} \right] + b_s V_s$$

and

$$(5.53b) \quad Q_{o,p} = \left[\frac{a_s - c + b_s V_s}{b_s + d} \right]$$

The model is illustrated graphically in Figure 5.2. The initial equilibrium solution is given by (P_o, Q_o) . Recall from Section 4.2 of the previous Chapter that

$$(4.5a) \quad P_o = a_s - b_s \left[\frac{a_s - c}{b_s + d} \right]$$

and

$$(4.5b) \quad Q_o = \frac{a_s - c}{b_s + d}$$

Generic promotion in an individual market segment or segments induces the aggregate demand shift, V_s . The increase in producer surplus which results from this activity can be represented by $P_o X Y P_{o,p}$, which is the hatched area in Figure 5.2.

By adapting equation (4.34a) in the previous Chapter, this change in producer surplus can be incorporated into the algebraic model. Now, the increase in producer surplus, ΔPS , is given by

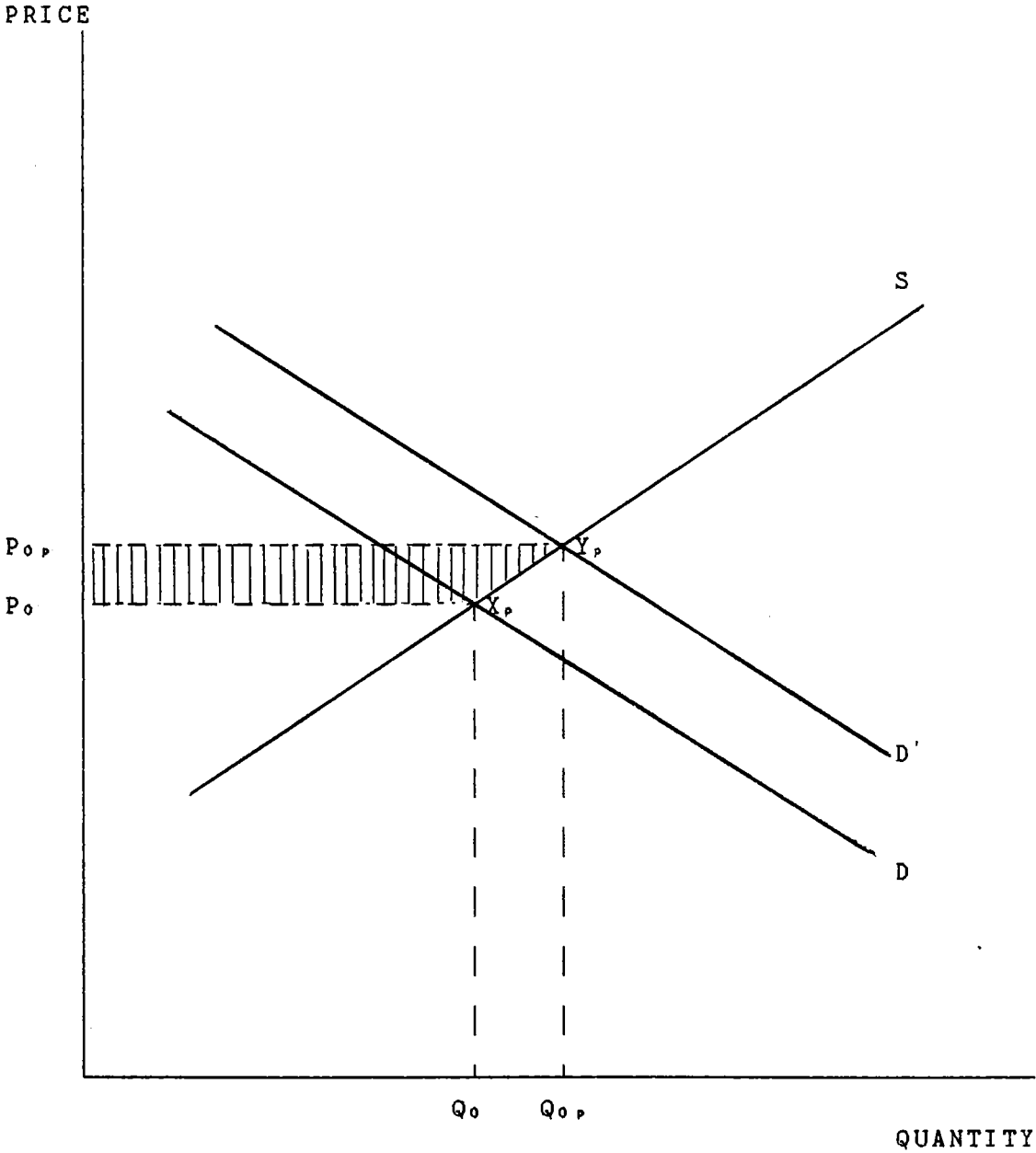


FIGURE 5.2: PRODUCER SURPLUS GAINS FROM GENERIC PROMOTION
UNDER COMPETITIVE PRICING

$$(5.54) \quad \Delta PS = \Delta TR - \Delta C$$

where ΔTR is the change in total revenue

and ΔC is the opportunity cost of resources associated with the increased output, $(Q_{0,p} - Q_0)$.

The change in total revenue, TR , is given by

$$(5.55a) \quad \Delta TR = P_{0,p} Q_{0,p} - P_0 Q_0$$

By substituting equations (5.53a), (5.53b), (4.5a) and (4.5b) into equation (5.55a), the following equation emerges.

$$(5.55b) \quad \Delta TR = \frac{b_a V_a}{b_a + d} \left[2a_a - c - 2b_a \left[\frac{a_a - c}{b_a + d} \right] + b_a V_a \left[1 - \frac{b_a}{b_a + d} \right] \right]$$

The opportunity cost of additional resources used to produce the increased output associated with promotion, ΔC , is given by

$$(5.56a) \quad \Delta C = \int_{Q_0}^{Q_{0,p}} (c + dQ) dQ$$

That is,

$$(5.56b) \quad \Delta C = c(Q_{0,p} - Q_0) + \frac{d}{2} (Q_{0,p}^2 - Q_0^2)$$

Once again, by substituting equations (5.53b) and (4.5b) into equation (5.56b), the following expression results

$$(5.56c) \quad \Delta C = \frac{b_a V_a}{b_a + d} \left[c + \frac{d(a_a - c)}{b_a + d} + b_a V_a \frac{d}{2(b_a + d)} \right]$$

Therefore, by subtracting equation (5.56c) from equation (5.55b), producer surplus gains from generic

promotion under competitive pricing, ΔPS , can be calculated as follows.

$$(5.57) \Delta PS = \frac{b_a V_a}{b_a + d} \left[2(a_a + c) - \left[\frac{a_a - c}{b_a + d} \right] [2b_a + d] \right. \\ \left. + b_a V_a \left[\frac{d}{2(b_a + d)} \right] \right]$$

The above expression will not be transformed to elasticities as was done in the previous Chapter, since the resulting equations are more cumbersome than illuminating.

In summary, when a generic promotion effect occurs in individual market segments, the result is a demand shift in the appropriate segment. The influence of such demand shifts on producer surplus under competitive pricing assumptions is given in equation (5.57). This expression will be used as a benchmark in later Sections of this study when comparisons are made on the relative returns from this type of promotion under discriminatory pricing.

5.4.3 A Discriminatory Pricing Model with Generic Promotion

Having established the competitive pricing benchmark in the previous Section, the next step is to construct a model with the same promotional effects, but under

discriminatory pricing assumptions. Such a model is outlined below.

As with the competitive pricing model, assume that a fixed amount of generic promotion in market segment i increases demand by V_i at all price levels. That is, recall equation (5.50a).

$$(5.50a) \quad P_{i,p} = a_i - b_i Q_i + b_i V_i$$

where Q_i is output in market segment i

V_i is the increase in demand after generic promotion in market segment i

and $P_{i,p}$ is the price in market segment i after generic promotion

Alternatively,

$$(5.50b) \quad P_{i,p} = a_i' - b_i Q_i$$

where $a_i' = a_i + b_i V_i$

By recalling the discriminatory pricing model developed in the previous Chapter, average revenue in the aggregate market after generic promotion and assuming discriminatory pricing, $P_{a,p}$, is given by

$$(5.58) \quad P_{a,p} = a_a' - b_a Q_a + \frac{k'}{Q_a}$$

where Q_a is aggregate output

$a_a' = a_a + b_a V_a$

with $V_a = \sum_{i=1}^n V_i$

$$\text{and } k' = \sum_{i=1}^n \left[\frac{\{(a_i - a_a) + (b_i V_i - b_a V_a)\}^2}{4b_i} \right]$$

Recall once again the supply curve.

$$(4.3) \quad P_s = c + dQ; \quad d > 0, \quad c < a_s$$

where P_s and Q are supply price and output respectively.

Therefore, after generic promotion under discriminatory pricing, equilibrium is given by

$$(5.59) \quad P_{d1p} = P_s$$

This yields the following equilibrium price, P_{d1p} , and output, Q_{1p} .

$$(5.60a) \quad P_{d1p} = c + d \left[\frac{a_s' - c + X'^{0.5}}{2(b_s + d)} \right]$$

where $X' = (a_s' - c)^2 + 4k'(b_s + d)$

and other variables are as defined above.

$$(5.60b) \quad Q_{1p} = \frac{a_s' - c + X'^{0.5}}{2(b_s + d)}$$

where variables are as previously defined.

The model is illustrated in Figure 5.3. The initial price-discriminating equilibrium solution is given by (P_{d1}, Q_1) . Recall from the previous Chapter that

$$(4.24a) \quad Q_1 = \frac{a_s - c + X^{0.5}}{2(b_s + d)}$$

An alternative formulation of P_{d1} is given by

$$(4.23c) \quad P_{d1} = c + d \left[\frac{a_s - c + X^{0.5}}{2(b_s + d)} \right]$$

where, in both cases, variables are as previously defined.

Therefore, after the demand shift, V_a , the increase in producer surplus from generic promotion under discriminatory pricing is given by $P_{d1}X_dY_dP_{d1p}$, which is the hatched area in Figure 5.3.

Recall equation (5.54) which gives producer surplus, ΔPS .

$$(5.54) \quad \Delta PS = \Delta TR - \Delta C$$

where ΔTR is the change in total revenue

and ΔC is the opportunity cost of resources associated with the increased output, $(Q_{1p} - Q_1)$.

This change in producer surplus, ΔPS , can be evaluated in a similar manner to that used in the previous Section. That is,

$$(5.61a) \quad \Delta TR = P_{d1p}Q_{1p} - P_{d1}Q_1$$

where variables are as previously defined.

Substituting equations (5.60a), (5.60b), (4.24a) and (4.23c) into this gives

$$(5.61b) \quad \Delta TR = \frac{c_a(a'_a - a_a) + c_a(X'^{0.5} - X^{0.5}) + 2d(k' - k)}{2(b_a + d)}$$

$$+ \frac{2d \left[[(a_a'^2 - a_a^2) - 2c(a'_a - a_a)] + [(a'X'^{0.5} - aX^{0.5})ctd - c(X'^{0.5} - X^{0.5})] \right]}{4(b_a + d)^2}$$

where variables are as previously defined.

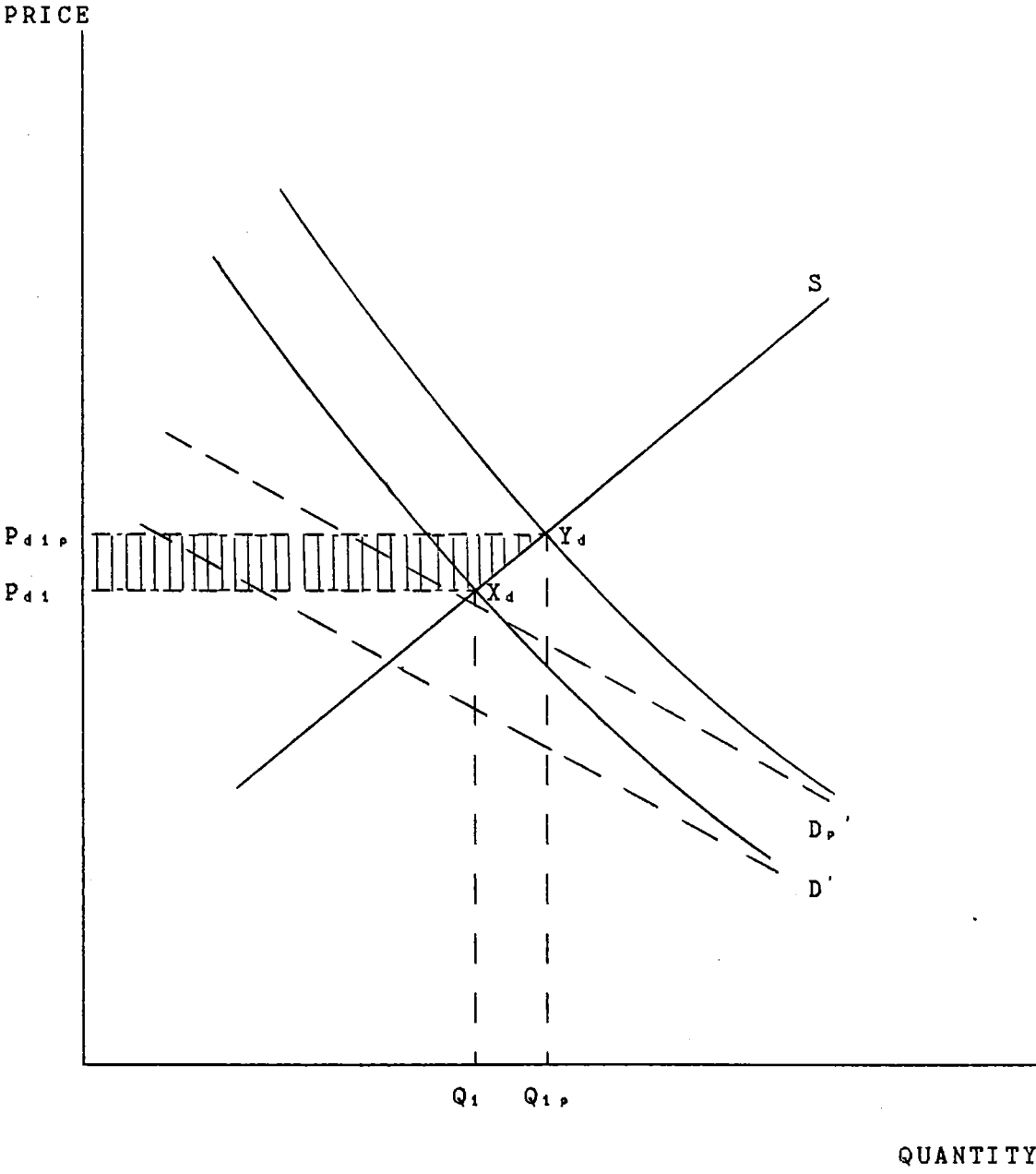


FIGURE 5.3: PRODUCER SURPLUS GAINS FROM GENERIC PROMOTION
UNDER DISCRIMINATORY PRICING

Similarly,

$$(5.62a) \quad \Delta C = c(Q_{1p} - Q_1) + \frac{d}{2} (Q_{1p}^2 - Q_1^2)$$

Substituting (5.60b) and (4.24a) into this gives

$$(5.62b) \quad \Delta C = \frac{c(a_a' - a_a) + c(X'^{0.5} - X^{0.5}) + d(k' - k)}{2(b_a + d)} \\ + \frac{d[(a_a'^2 - a_a^2) - 2c(a_a' - a_a) + (a_a' X'^{0.5} - a_a X^{0.5}) - c(X'^{0.5} - X^{0.5})]}{4(b_a + d)^2}$$

Therefore, subtracting (5.62b) from (5.61b) gives

$$(5.63) \quad \Delta PS = \frac{d \left[[2(b_a + d)(k' - k)] + [(a_a'^2 - a_a^2) - 2c(a_a' - a_a)] ctd \right. \\ \left. + [(a_a' X'^{0.5} - a_a X^{0.5}) - c(X'^{0.5} - X^{0.5})] \right]}{4(b_a + d)^2}$$

To conclude, a generic promotion effect in individual market segments results in a demand shift in the appropriate segments. The influence of these demand shifts on producer surplus under discriminatory pricing assumptions is given in equation (5.63). Comparison of this equation with the measure for increased producer surplus which results from identical demand shifts under competitive pricing assumptions allows some assessment to be made of the relative returns from this type of promotion under alternative pricing regimes.

5.4.4. Conclusion

Since it was not possible to incorporate optimal promotion expenditure in alternative segments into a model

which showed the actual magnitude of producer returns from such promotion given optimal rather than competitive pricing, models were constructed which allowed such returns to be calculated from a given demand shift. It must be acknowledged however, that the models abstract from the magnitude of advertising expenditure required to produce such demand shifts, although such a mechanism was investigated.

In Chapter 6, these models will be utilized to determine whether returns from promotion expenditure result in greater gains for producers under optimal pricing than they do under uniform pricing.

5.5 THE EFFECT OF PRODUCT DIFFERENTIATION ON PRODUCER SURPLUS UNDER DISCRIMINATORY PRICING

In addition to a generic promotion effect in individual market segments, the second conceptual promotion effect which has been noted in the literature occurs when a product can be differentiated from that of a competitor. Such product differentiation has the effect of making demand for a product less elastic than it might otherwise be (de Boer, 1977; May, 1977). Such an effect can be incorporated into the price discrimination and competitive pricing models developed in the previous Chapter.

Recall equation (4.28c), which gives the export demand elasticity facing a marketing institution in a particular market segment, η_{x1a} .

$$(4.28c) \quad n_{xia} = n_i \phi_{ia} \frac{C_i}{X_{ia}} - \left[e_i \theta_{ia} \frac{S_i}{X_{ia}} + \sum_{j=1}^n e_{ij} \theta_{ja} \frac{X_{ij}}{X_{ia}} \right]$$

where n_i is the own price elasticity of demand in i

e_i is the own price elasticity of supply in i

e_{ij} are the appropriate cross elasticities of supply

$\frac{X_{ia}}{C_i}$ is a 's market share in i

$\frac{X_{ia}}{S_i}$ and $\frac{X_{ia}}{X_{ij}}$ are measures of a 's importance as

a supplier to i

and ϕ_{ia} , θ_{ia} and θ_{ja} are measures of the appropriate price transmission elasticities.

For simplicity, these price transmission elasticities can be assumed equal, and equation (4.28c) then becomes

$$(4.28d) \quad n_{xia} = \phi_{ia} \left[n_i \frac{C_i}{X_{ia}} - \left[e_i \frac{S_i}{X_{ia}} + \sum_{j=1}^n e_{ij} \frac{X_{ij}}{X_{ia}} \right] \right]$$

where variables are as previously defined.

Now when price transmission is perfect, ϕ_{ia} is 1, and when no price transmission occurs, ϕ_{ia} is zero. Therefore, when country a 's product is differentiated from that of its competitors, ϕ_{ia} falls since price transmission becomes less perfect. From (5.28d), it becomes obvious that as ϕ_{ia} falls, the export demand elasticity, n_{xia} , also falls. Therefore, the competitive supply response is reduced.

Therefore, the effect of differentiating a product from that of a competitor in any or all market segments can be ascertained through its effect on the appropriate demand elasticities. This in turn influences the magnitude of producer surplus under discriminatory pricing. Competitive returns will obviously remain unchanged, since altering the elasticity of demand in individual market segments will not alter the competitive equilibrium solution. This implies that purely product-differentiating promotion makes little sense under competitive pricing procedures.

Within the agricultural economics literature, there appear to be no studies which specifically deal with linkages between advertising expenditure and changes in price transmission elasticities. As a consequence, the influence on producer returns of given changes in demand elasticities, Δn_i , which result from product-differentiating promotion will be investigated. This allows the relative effect of such advertising in alternative market segments under discriminatory pricing to be determined. However, as with the generic promotion case, it must be borne in mind that such an analysis abstracts from the advertising expenditure required to produce these changes in demand elasticities.

5.6 SUMMARY AND CONCLUSIONS

The broad objective of this Chapter was to investigate aspects of promotion in multiple market segments where a

supply response by producers occurs in response to any increased price which they receive from this promotion activity.

It was ascertained that optimal decision rules for promotion in multiple market segments under these supply conditions had not been developed under either competitive or discriminatory pricing procedures. As a consequence, such decision rules were developed by appropriate extensions to the Nerlove-Waugh theorem of optimal advertising for an aggregate market with supply response.

The first such extension incorporated uniform pricing across market segments, and the following four conclusions emerged from the model.

- (a) The optimal aggregate advertising intensity is identical to that under the Nerlove-Waugh theorem, with the exception that the advertising demand elasticity for the Nerlove-Waugh aggregate market case is replaced by the sum of the advertising elasticities in individual market segments weighted by the share of output allocated to these segments.
- (b) The optimal allocation of advertising expenditure to an individual market segment is given by the ratio of the advertising elasticity of demand in that segment weighted by the share of output allocated to that segment to the sum of all such weighted averages.
- (c) The optimal ratio of advertising to sales generated in a particular market segment is

identical to that for the Nerlove-Waugh theorem with the exception that the aggregate market Nerlove-Waugh advertising elasticity is replaced by the advertising elasticity in the relevant market segment.

- (d) The ratio of optimal advertising per unit sales in market segment i to that in market segment j is equivalent to the ratio of the corresponding advertising elasticities of demand.

The second extension to the Nerlove-Waugh theorem incorporated discriminatory pricing between market segments. Three conclusions emerged from this model.

- (e) The formula for the optimal aggregate advertising intensity in a particular market segment is similar to that which emerges under the aggregate Nerlove-Waugh theorem. However, it includes an additional term in the denominator which is a function of the price elasticity of demand in the market segment in question, the price elasticity of supply, and the ratio of the pool price to the optimal price in the market segment.
- (f) For a two-segment case, the optimal advertising intensity in the less price elastic segment is greater than it would be under Nerlove-Waugh prescriptions. Conversely, for the more price elastic market segment, this intensity would be correspondingly less than it would be under Nerlove-Waugh assumptions.

(g) In the two-market segment case once again, relatively more advertising effort is directed to the less price elastic market segment under discriminatory pricing than would be the case under competitive pricing across market segments. Conversely, relatively less advertising effort is directed towards the more price elastic segment.

The above decision rules enhance our general understanding of how advertising effort might be more profitably allocated under market assumptions which are more realistic than those conventionally employed to prescribe the direction of advertising effort in typically structured agricultural industries.

However, it was not possible to incorporate such formulae into a marketing mix model which indicates returns to producers from optimal pricing and promotion policies. Therefore, the influence of given, rather than optimal, promotion effects on producer returns was considered.

In order to do this, it was necessary to incorporate promotion effects into the competitive and discriminatory pricing models previously developed. To facilitate this, two conceptual promotion effects were considered. The first, termed a 'generic' promotion effect, refers to the ability of a marketing agency to increase demand for its product at all price levels in a particular market segment. The second effect considered was a 'product-differentiating' effect, which occurs when a marketing agency manages to alter its demand curve in a particular

segment by differentiating its product from that of its competitors.

Generic promotion effects were incorporated into the competitive and discriminatory pricing models constructed in Chapter 4. This allowed expressions to be derived for the changes in producer surplus which result from this type of promotion in selected market segments under each of these pricing regimes. Similarly, product-differentiating promotion effects can be incorporated into the discriminatory pricing model through an appropriate adjustment to the demand elasticities facing the marketing institution.

In Chapter 6, the formulae emerging from these gains models will be evaluated, and numerical simulations will be conducted to determine the relative influence of promotion in alternative market segments under alternative pricing regimes.

CHAPTER SIX

THE RELATIONSHIP BETWEEN MARKET CHARACTERISTICS AND THE OUTCOMES OF MARKET SEGMENTATION POLICIES

6.1 INTRODUCTION

In the previous two Chapters, the operation of market segmentation policies in typically structured agricultural industries has been modelled. Expressions have been derived which can now be used to evaluate the impact of market characteristics on the outcomes of market segmentation policies.

This Chapter uses these relationships to isolate those market characteristics which are of significance, and to explore the sensitivity of market segmentation outcomes to changes in these variables.

In Section 6.2, such variables are isolated and a numerical example which can be used as a basis for simulation exercises is developed. Section 6.3 considers those variables which influence the short-run outcomes of such segmentation policies, while variables affecting long-run returns are considered in Section 6.4.

These long-run variables are, however, influenced by a further set of variables, which are investigated in Section 6.5. Since most discussion has concentrated on the influence of market segmentation policies on producer returns, the secondary producer objective of reducing

producer price variability will be considered in Section 6.6.

Finally, in Section 6.7, the market segmentation model is applied to the New Zealand sheepmeats industry. This allows the actual magnitude of returns which may be expected in a particular industry to be assessed.

6.2 BASIC MODEL FEATURES

6.2.1 Variables Influencing Segmentation Outcomes

Market segmentation policies have both a short-run and a long-run impact on the primary producer objective of increasing producer returns.

The short-run effects were modelled in Chapter 4, and are represented by the immediate returns, k , from reallocating a given output according to discriminatory pricing principles. This was given by the expression

$$(4.22) \quad k = \frac{1}{2} P_0 Q_0 \sum_{i=1}^n \left[s_i n_i \left[\frac{1}{n_i} - \frac{1}{n_a} \right]^2 \right]$$

where Q_0 is the given output

P_0 is the competitive price at Q_0

s_i is the proportion of total output allocated to market segment i under competitive pricing
 n_i and n_a are the price elasticities of demand (absolute value) in market segment i and the aggregate market, respectively, at the competitive price, P_0 .

The longer-run impact on producer returns of market segmentation policies as measured by producer surplus

gains, ΔPS , was encapsulated in expression (4.34c) which incorporates the appropriate supply responses to discriminatory pricing.

$$(4.34c) \Delta PS = \frac{1}{4e} \left[\frac{2k}{\left[\frac{1}{n_a} + \frac{1}{e} \right]} - P_0 Q_0 + \left[P_0^2 Q_0^2 + 4k P_0 Q_0 \frac{1}{\left[\frac{1}{n_a} + \frac{1}{e} \right]} \right]^{0.5} \right]$$

where e is the price elasticity of supply at (P_0, Q_0)

and other variables are as defined above.

In addition, expressions were derived for the various total revenue components of this producer surplus change, and the own supply response to altered pricing policies.

Effectively, the long-run generic promotion elements in a market segmentation policy were incorporated into the discriminatory pricing model by intercept shifts in the demand equations. That is,

$$(6.1) \quad a_i' = a_i + b_i V_i$$

where V_i represents the increase in demand at all price levels.

Expressions for increased producer returns which resulted from generic promotion were then derived.

Similarly, any long-run product-differentiating promotion effects in individual market segments were incorporated through a change in the price elasticity of demand, n_i , facing an institution for its product in individual market segments.

However, some of the variables which appear in the above expressions are themselves influenced by other

variables. For example, the price elasticity of demand in the aggregate market, n_a , can be calculated from the corresponding elasticities in the individual market segments, n_i , and the share of output allocated to these market segments, s_i . That is,

$$(6.2) \quad n_a = \sum_{i=1}^n n_i s_i$$

Similarly, the long-run price elasticity of demand facing an agency for its product in individual market segments, n_i , can be calculated by amending equation (4.28c) to give

$$(6.3) \quad n_i = \phi_i \left[n_{i1} \frac{C_i}{X_{i,a}} - e_{i1} \frac{S_{i,a}}{X_{i,a}} \right]$$

where ϕ_i represents all consumer and producer price transmission elasticities which are assumed equal

n_{i1} is the market price elasticity of demand in market segment i

e_{i1} is the price elasticity of supply for competitors and is assumed to be equal for all supply sources

$\frac{C_i}{X_{i,a}}$ is the market share of the product held by C_i the marketing agency in country i

$\frac{S_{i,a}}{X_{i,a}}$ is the ratio of product supplied from all other sources to that supplied by the marketing agency.

The product-differentiating influence referred to previously is made explicit in this equation, since it

effectively alters the price transmission elasticities, ϕ_i .

Similarly, an expression which gives an approximation to the demand shift from generic promotion in any market segment, V_i , is given by

$$(5.50b) \quad V_i = \beta_i Q_i \frac{\Delta A_i}{A_i}$$

where A_i is the initial advertising level

Q_i is initial demand

ΔA_i is increased advertising expenditure

and β_i is the advertising elasticity of demand at (Q_i, A_i) .

From the above summary of the important model features, a list of variables which influence the outcomes of market segmentation policies can be derived. These variables can be classified according to whether they are influential in the short-run or the long-run.

The comparative static nature of the model developed implies that the incorporation of promotion effects is essentially a long-run phenomenon (Nerlove and Waugh, 1961). Therefore, the relevant short-run variables are those which determine the outcome of a short-run reallocation of a given output according to discriminatory pricing principles.

The appropriate long-run variables are those which influence the long-run model outcomes where promotion has been incorporated. However, in the interests of clarity, it is useful to consider those factors which influence the outcomes from long-run discriminatory pricing, and then to

consider variables which relate to long-run discriminatory pricing under various promotional strategies.

The basic influential variables are summarized in Table 6.1. In following Sections, each of these variables will be varied in turn, and the impact of this shock on model outcomes will be evaluated. In Table 6.2, the factors which in turn influence the long-run variables are also listed. These will likewise be varied to determine their influence on the magnitude of the appropriate long-run variables, and ultimately, on market segmentation outcomes.

6.2.2 A Base Model

In order to investigate the impact of the above variables, a basic two-segment version of the model which assigns specific values to the above variables has been developed. The numerical features of this base model are outlined in Table 6.3.

The objective of the simulation exercises is to determine the relative influence of particular variables on model outcomes, rather than to simulate the actual performance of market segmentation policies in a given industry. The above parameters do not purport to represent any specific industry, although they characterize agricultural conditions to the extent that this is possible. The base price and output parameters are essentially mid-range estimates from New Zealand's five major agricultural industries. A representative supply elasticity is difficult to choose, since this

TABLE 6.1
BASIC VARIABLES WHICH INFLUENCE THE OUTCOMES OF MARKET
SEGMENTATION POLICIES

=====		
SHORT-RUN:	n_i	Price elasticities of demand facing an agency in individual market segments
	s_i	Shares of output allocated to individual market segments
	P_0, Q_0	Initial equilibrium price and output
LONG-RUN:	The above variables	
	n_a	Price elasticity of demand facing an agency in the aggregate market
	e	Price elasticity of supply
	V_i	Demand shift from generic promotion in individual market segments
=====		

TABLE 6.2
FACTORS INFLUENCING THE MAGNITUDE OF LONG-RUN VARIABLES

=====			
VARIABLE		FACTORS	

n_i	n_{i1}	Market price elasticity of demand in market segment i	
	$\frac{X_{i1}}{C_i}$	Market share of product in market segment i	
	e_{i1}	Competitive supply elasticities in market segment i	
	ϕ_i	All price transmission elasticities with respect to market segment i	
V_i	Q_i	Initial level of output in market segment i	
	$\frac{\Delta A_i}{A_i}$	Proportionate increase in advertising expenditure in market segment i	
	β_i	Advertising elasticity of demand in market segment i	
		=====	

parameter is influenced by the nature of the product, alternative land-use possibilities, and the time horizon in question. In the base example, a longer-run perspective is focused on with some inelasticity in supply still remaining. There is some degree of arbitrariness in setting the base demand parameters, since these are likely to vary according to the type of the product and the dimension, whether spatial, temporal or product form, in which segmentation is undertaken.

6.3 VARIABLES INFLUENCING SHORT-RUN OUTCOMES

6.3.1 The Competitive Equilibrium Position

Short-run market segmentation outcomes are influenced by the magnitude of the competitive equilibrium price and output, P_0 and Q_0 . From equation (4.22), it is obvious that short-run producer returns vary directly and proportionately with these variables.

However, it may be more relevant to consider the relative, rather than the absolute, increase in producer returns; that is, the percentage increase in immediate total revenue, measured by $\frac{k}{P_0 Q_0}$. In this case, a given percentage increase in either the base price or the quantity would lead to the same percentage increase in producer returns.

6.3.2 Price Elasticities of Demand

The magnitude of the short-run producer gains, k , is influenced by the magnitude of the price elasticities of demand in the individual market segments. However, it is

TABLE 6.3
BASE MODEL PARAMETERS

=====	
P ₀	\$1000/t
Q ₀	250,000t
n ₁	0.5
n ₂	1.5
s ₁	0.5
s ₂	0.5
e	1.0
=====	

not obvious from equation (4.22) how such gains might vary when price elasticities in alternative market segments vary.

Numerical simulations on the base model were therefore conducted. In the first instance, the price elasticity in the relatively less price elastic segment, n_1 , was increased from 0.5 until it reached the value of n_2 , 1.5, and was then decreased from 0.5 until it was very low. The values of all other model variables were held constant during this exercise. The results of these simulations are presented in Table 6.4.

These indicate that when demand in both segments is isoelastic, short-run revenue gains are zero. As n_1 falls from this value, both the absolute value of the short-run revenue gains, k , and the relative increase in total revenue, $\Delta TR.(\%)$, increase at an increasing rate. In an attempt to explore this observation more closely, appropriate relationships between the individual market segment elasticities and the aggregate elasticity were noted and are shown in the last three columns of Table 6.4. The effect of the falling n_1 is to widen the divergence between the individual elasticities, $(n_2 - n_1)$, and to decrease the value of the aggregate elasticity, n_a . Obviously, this divergence as a proportion of the aggregate elasticity, $\left[\frac{n_2 - n_1}{n_a} \right]$, increases as n_1 falls.

TABLE 6. 4
THE INFLUENCE OF VARYING LEVELS OF n_1
ON SHORT-RUN PRODUCER RETURNS

n_1	$k(\text{\$M})$	$\Delta TR_s(\%)$	$(n_2 - n_1)$	n_s	$\left[\frac{n_2 - n_1}{n_s} \right]$
0. 125	193. 9	77. 6	1. 38	0. 81	1. 70
0. 250	74. 4	29. 8	1. 25	0. 88	1. 42
0. 500	20. 8	8. 3	1. 00	1. 00	1. 00
1. 000	2. 1	0. 8	0. 50	1. 25	0. 40
1. 250	0. 4	0. 1	0. 25	1. 38	0. 18
1. 500	0. 0	0. 0	0. 00	1. 50	0. 00

The demand elasticity in the relatively price elastic market, n_2 , was then varied in a similar manner. It was first decreased from its base value of 1.5 to the value of n_1 , 0.5, and then increased beyond its base value until it was very high. The results are presented in Table 6.5.

As n_2 increased, both absolute and relative short-run producer gains rose. This increase in gains coincided with increased values for the aggregate elasticity, n_a , the divergence between these elasticities, $(n_2 - n_1)$, and a relative measure of this divergence, $\left[\frac{n_2 - n_1}{n_a} \right]$.

A comparison of Tables 6.4 and 6.5 indicates that relatively large increases in producer gains are associated with reducing levels of n_1 , and hence, n_a . However, relatively smaller increases in these gains are associated with increasing levels of n_2 , and as a consequence, n_a . In both cases, the divergence between the individual elasticities increased.

These relationships between producer returns, the aggregate elasticity, and the divergence between individual elasticities will now be investigated more precisely. In the first instance, both demand elasticities, n_1 and n_2 , are varied above and below their base values in a manner which holds the absolute divergence between them constant. This effectively varies the aggregate elasticity. These simulations are presented in Table 6.6, and indicate that, for a given divergence, producer returns decrease as the value of the aggregate elasticity, n_a , increases. This implies that the ability to increase total revenue through exploiting demand in the

TABLE 6.5

THE INFLUENCE OF VARYING LEVELS OF n_2
ON SHORT-RUN PRODUCER RETURNS

n_1	$k(\$M)$	$\Delta TR_s(\%)$	$(n_2 - n_1)$	n_s	$\left[\frac{n_2 - n_1}{n_s}\right]$
0.5	0.0	0.0	0.0	0.50	0.00
1.0	10.4	4.2	0.5	0.75	0.67
1.5	20.8	8.3	1.0	1.00	1.00
2.5	33.3	13.3	2.0	1.50	1.33
5.0	46.0	18.4	4.5	2.75	1.64
10.0	53.7	21.5	9.5	5.25	1.81

relatively less price elastic segment is greater when monopoly power in that segment, and in the total market itself, is greater.

Individual segment elasticities were then varied in an alternative way. In this case, the divergence between n_1 and n_2 was varied in a manner which held the aggregate elasticity, n_a , constant. Table 6.7 illustrates these results. In this case, for a given value of n_a , producer returns increase as the divergence between individual elasticities, $(n_2 - n_1)$, increases.

In conclusion, therefore, short-run producer returns will be relatively large in both absolute and relative terms, when, at the competitive equilibrium, the divergence between demand elasticities in market segments, $(n_2 - n_1)$, is relatively large, and these elasticities are such that the aggregate demand elasticity, n_a , is relatively low. Since the price elasticity of demand for agricultural products is known to be relatively low, it seems obvious that the application of market segmentation policies might appear superficially attractive to marketing agencies operating in these industries.

6.3.3 Shares of Output Allocated to each Segment

The final variable which influences the short-run returns from market segmentation policies will now be considered. This is the share of output allocated to individual market segments under competitive allocation procedures, and is denoted by s_i . From equation (4.22),

TABLE 6.6
THE INFLUENCE OF VARYING LEVELS OF n_1 AND n_2 ,
WITH CONSTANT DIVERGENCE, $(n_2 - n_1)^1$,
ON SHORT-RUN PRODUCER RETURNS

n_1	n_2	$k(\$M)$	$\Delta TR_s(\%)$	n_a
0.25	1.25	66.7	27.0	0.75
0.50	1.50	20.8	8.3	1.00
0.75	1.75	9.5	3.8	1.25
1.00	2.00	5.2	2.1	1.50
1.50	2.50	2.1	0.8	2.00
2.50	3.50	0.6	0.2	3.00
5.00	6.00	0.1	0.0	5.50

1. The divergence, $(n_2 - n_1)$ is assumed to equal its base value, 1.0.

TABLE 6.7
THE INFLUENCE OF VARYING LEVELS OF n_1 AND n_2 ,
WITH CONSTANT n_a^1 , ON SHORT-RUN PRODUCER RETURNS

n_1	n_2	$k(\$M)$	$\Delta TR, (\%)$	$(n_2 - n_1)$
0.13	1.88	194.8	77.9	1.75
0.25	1.75	80.3	32.1	1.50
0.50	1.50	20.8	8.3	1.00
0.75	1.25	4.2	1.7	0.50
1.00	1.00	0.0	0.0	0.00

1. The aggregate elasticity, n_a , is assumed to equal its base value, 1.0.

it is not obvious how producer returns vary with changes in this parameter. Therefore, a range of simulations were undertaken. The value of s_1 was initially decreased from its base value of 0.5 and then increased beyond this. The value of s_2 was correspondingly varied. The results of this simulation are shown in Table 6.8.

This shows that when the share of output allocated to the less price elastic segment under competitive pricing, s_1 , falls, then the increase in producer returns, both in absolute and relative terms, also falls. Note that this fall is associated with increasing values of the aggregate demand elasticity, n_a .

However, when s_1 rises beyond its base value, thereby inducing a fall in n_a , producer returns do not increase beyond the base level but remain static and then decrease as s_1 rises and s_2 consequently falls.

A possible explanation for this result might be that as s_1 continues to rise, the more elastic segment is dwarfed by the inelastic segment in this particular example. A reallocation of output from the inelastic to the elastic segment will increase total revenue from the inelastic segment. However, the increased output in the relatively elastic segment may be so great relative to the size of the segment, that it results in demand in that segment moving from a price elastic situation to a price inelastic situation. As a result, total revenue in this segment may not increase to the extent which might be expected if output moved from one price elastic point to another price elastic point on the demand curve.

TABLE 6.8
THE INFLUENCE OF VARYING PROPORTIONS OF OUTPUT
ALLOCATED TO INDIVIDUAL SEGMENTS, s_1 AND s_2 ,
ON SHORT-RUN PRODUCER RETURNS

s_1	s_2	$k(\text{\$M})$	$\Delta TR_1(\%)$	n_1
0.125	0.875	6.6	2.7	1.375
0.250	0.750	12.5	5.0	1.250
0.500	0.500	20.8	8.3	1.00
0.750	0.250	20.8	8.3	0.750
0.875	0.125	14.6	5.8	0.625

On the basis of these results, it would not seem wise to draw firm conclusions on the relationship between the observed share of output in individual market segments before market segmentation and the magnitude of producer gains which result from market segmentation.

6.3.4 Conclusion

In this Section, variables which influenced short-run returns from market segmentation outcomes were analyzed using the linear model which was developed. A number of conclusions were drawn on the basis of either analytical or numerical observation.

Firstly, short-run revenue returns will be high, in absolute terms, when the size of the industry is large before price discrimination, and the corresponding competitive price is high. However, in relative terms, returns from market segmentation do not vary with respect to these variables.

In addition, in both absolute and relative terms, short-run returns will be large when, in competitive equilibrium, the aggregate demand elasticity is low, and the divergence between individual demand elasticities is large.

6.4 VARIABLES INFLUENCING LONG-RUN OUTCOMES

6.4.1 The Aggregate Price Elasticity of Demand

The short-run variables considered above influence the long-run returns from market segmentation in a similar

manner to that already described. However, in the long-run, the individual elasticities, n_i , are themselves influenced by a further set of variables, which will be discussed in the next Section.

Despite the fact that the aggregate elasticity, n_a , is itself composed of some of the variables discussed above, its influence on long-run producer returns will be evaluated. This is because equation (4.34c) indicates that this composite elasticity influences these returns directly, as well as through the short-run returns, k , which have already been considered.

In doing this, the divergence between the individual demand elasticities was assumed to remain constant as the aggregate elasticity initially fell from its base value of 1.0 and then rose from this base value until it was relatively high. The various total revenue components of producer surplus were calculated, as was the long-run own supply response by producers. The results of this exercise are presented in Table 6.9.

As the aggregate demand elasticity, n_a , rises, then for a given divergence between segment elasticities, $(n_2 - n_1)$, long-run producer surplus returns fall. Consider the total revenue component of these long-run producer surplus returns, ΔTR . This consists of short-run revenue returns, k , and the long-run quantity adjustment to total revenue, discussed in Chapter 4, which can be calculated as $(\Delta TR - k)$.

As noted previously, this short-run component, k , decreases as the aggregate demand elasticity increases.

The quantity adjustment to this short-run revenue may be either negative or positive. When the aggregate demand elasticity is price inelastic at the competitive equilibrium then this adjustment will be negative and dissipate short-run gains. On the other hand, when this elasticity (as measured on the demand curve after supply response) is greater than one, this adjustment will be positive, and therefore enhance short-run gains. As the aggregate demand elasticity rises, this quantity adjustment becomes a relatively more important component of total revenue gains.

Producer surplus gains, ΔPS , demonstrate a similar pattern with respect to the aggregate demand elasticity, with returns becoming rapidly smaller as demand becomes more elastic. As a result, the supply response, ΔQ , also declines as the demand elasticity rises, since increased returns from market segmentation policies are correspondingly lower.

6.4.2 The Price Elasticity of Supply

A variable which is of some importance in determining the magnitude of the own supply response to discriminatory pricing is the own price elasticity of supply.

It is obvious from the previously calculated total revenue equation (4.32d), that long-run revenue gains are equal to the short-run gains, k , when the supply elasticity is zero. That is,

$$(6.4) \quad \Delta TR = k \quad \text{when } e = 0.$$

TABLE 6.9

THE INFLUENCE OF THE AGGREGATE DEMAND ELASTICITY, n_a ,
ON LONG-RUN PRODUCER RETURNS

n_1	n_2	n_a	k	ΔTR	ΔPS	ΔQ
			(\$M)	(\$M)	(\$M)	('000t)
0.01	1.01	0.51	3,033.4	1,420.8	710.4	396.0
0.50	1.50	1.00	20.8	20.4	10.2	10.0
1.00	2.00	1.50	5.2	6.2	3.1	3.1
1.50	2.50	2.00	2.1	2.8	1.4	1.4
2.00	3.00	2.50	1.0	1.5	0.7	0.7
4.50	5.50	5.00	0.1	0.2	0.1	0.1
9.50	10.50	10.00	0.0	0.0	0.0	0.0

It is not intuitively obvious how these revenue gains behave as supply becomes more elastic, although an analytical expression, equation (6.5), can be derived which shows the limiting value of ΔTR as e approaches infinity.

$$(6.5) \quad \Delta TR = \left[\frac{P_0^2 Q_0^2}{4} + P_0 Q_0 k n_s \right]^{0.5} - \frac{P_0 Q_0}{2} \quad \text{when } e \rightarrow \infty$$

Proofs to equations (6.4) and (6.5) are given in Appendix 3.

Unfortunately, it is not obvious from equation (4.34c) how producer surplus gains vary with the price elasticity of supply. However, intuition would suggest that such returns would range from the short run revenue gains, k , when the price elasticity of supply is zero, to zero when the price elasticity of supply approaches infinity. Recall from the discussion on producer surplus in Chapter 3, however, that this concept is only meaningful in the long-run when some inelasticity remains in the supply of one factor.

Once again, a series of appropriate simulations were run. The results, illustrated in Table 6.10, clarify the above relationships between producer surplus returns and the supply elasticity.

Obviously, short-run revenue gains, k , are constant with respect to the magnitude of the supply elasticity. However, for the base example illustrated in Table 6.10, the quantity adjustment revenue effect is negative, and increases (becomes more negative) as the supply elasticity

risers. This is observable through total revenue gains, ΔTR , which decrease, but not markedly so, as supply becomes more elastic. By contrast, producer surplus gains, which are equivalent to k when supply is completely inelastic, are rapidly eroded as supply becomes more elastic. Similarly, the supply response by producers increases markedly under these conditions.

6.4.3 Demand Shifts from Generic Promotion

As noted previously, the demand shift from generic promotion in an individual market segment, V_i , was treated as an intercept shift in this market segment. Consequently, the competitive pricing and discriminatory pricing models were algebraically reworked to include such an effect. Unfortunately, it was not obvious from these amended models (equations (5.57) and (5.63)), how generic promotion in individual market segments influences producer returns under discriminatory pricing relative to the benchmark competitive pricing. Consequently, such an effect was numerically simulated under each pricing regime, and a comparison made of the outputs from each of these models.

It was assumed that a five percent increase in total demand occurred at the competitive price as a result of generic promotion. That is, V_a is 12,500 tonnes in the base example. Demand shifts of alternative magnitudes have correspondingly proportional influences on model outcomes and therefore, will not be considered further.

TABLE 6.10
THE INFLUENCE OF THE SUPPLY ELASTICITY, e ,
ON LONG-RUN PRODUCER RETURNS

e	k	ΔTR	ΔPS	ΔQ
	(\$M)	(\$M)	(\$M)	('000t)
0.01	20.8	20.8	20.6	0.2
0.10	20.8	20.8	18.9	1.9
0.25	20.8	20.7	16.5	4.1
0.50	20.8	20.6	13.7	6.8
1.00	20.8	20.4	10.2	10.0
1.50	20.8	20.3	8.1	11.9
2.50	20.8	20.0	5.8	14.1
5.00	20.8	19.8	3.4	16.3
10.00	20.8	19.6	1.8	17.7

In the first instance, the entire shift was assumed to occur in segment 1, whereas in the second case, it was assumed to occur in segment 2. A third possibility, where half of the demand increase occurs in each market segment, was also considered. The results are presented in Table 6.11. In this Table, note that P' , Q' , s_1' and s_2' refer to the price, quantity and share of output allocated to market segments after generic promotion in the appropriate segment. Similarly, $\Delta TR'$ and $\Delta PS'$ refer to increased total revenue and increased producer surplus, respectively, from generic promotion given the appropriate pricing regime.

Consider the influence of generic promotion under competitive pricing. When this occurs in market segment 1 alone, both price and output increase with the result that both total revenue and producer surplus also increase. In addition, the share of this increased output allocated to segment 1 increases, while that in segment 2 decreases. When an alternative demand shift is assumed in segment 2 alone, the same increase in price, output, total revenue and producer surplus is observed. However, unlike the first demand shift considered, the share of output allocated to segment 1 decreases, while that to segment 2 increases when generic promotion occurs in this latter segment. The final promotion possibility considered allocated half of the postulated demand increase to each market segment. In this case, the same increases in price, output and producer returns occurred as with all

	V_1 ('000t)	V_2 ('000t)	V_m ('000t)	k (\$M)	ΔTR^* (\$M)	ΔPS^* (\$M)	P^* (\$'000/t)	Q^* ('000t)	s_1^*	s_2^*
COMPETITIVE PRICING	0.00	0.00	0.00	0.0	0.0	0.0	1.000	250	0.50	0.50
	12.50	0.00	12.50	0.0	12.6	6.3	1.025	256	0.53	0.47
	0.00	12.50	12.50	0.0	12.6	6.3	1.025	256	0.48	0.52
	6.25	6.25	12.50	0.0	12.6	6.3	1.025	256	0.50	0.50
DISCRIMINATORY PRICING	0.00	0.00	0.00	20.8	0.0	0.0	1.040	260	0.37	0.63
	12.50	0.00	12.50	27.5	19.1	9.5	1.076	269	0.38	0.62
	0.00	12.50	12.50	27.5	10.6	5.3	1.060	265	0.36	0.64
	6.25	6.25	12.50	23.0	14.7	7.3	1.068	267	0.37	0.63

TABLE 6.11: THE INFLUENCE OF GENERIC PROMOTION ON LONG-RUN PRODUCER RETURNS

previous cases, while the original share of output allocated to individual market segments is preserved.

By comparison, consider the case where discriminatory pricing is practised and generic promotion is undertaken in each market segment in turn. When this promotion is undertaken in the first segment alone, the pool price and output both increase, as does the short-term revenue gain, k , total revenue, and producer surplus. Note that, in this case, k measures the increase in short-run revenue from the original competitive pricing equilibrium before promotion to the discriminatory pricing equilibrium after promotion. Similar effects occur when promotion occurs in the second segment alone, although these are relatively dampened in this case. Once again, the case is considered where half the demand increase is assumed to occur in each segment. The result of this simulation gives values for variables which lie between those noted for the segment 1 and segment 2 examples.

When comparing the relative returns to producers under alternative pricing regimes, it is of interest to note that generic promotion in the relatively less price elastic segment leads to greater increases in producer revenue and producer surplus under discriminatory pricing than under competitive pricing. However, when promotion is undertaken in the relatively price elastic segment, this observation is reversed with greater gains being captured under competitive pricing.

This implies that care should be taken when allocating a promotion budget under discriminatory pricing, since

returns are relatively high when generic promotion is undertaken in the inelastic segment. However, when an identical absolute demand shift occurs in the elastic segment, producer returns are much lower. In fact, these simulations suggest that producer returns may actually be depressed by generic promotion in the relatively price elastic segment.

These results may be explained as follows. When generic promotion induces a parallel rightward shift in (linear) demand in a particular segment, then at each price, demand in this segment is relatively less price elastic than it was before promotion.

Therefore, generic promotion in the less price elastic segment alone further decreases this price elasticity. This, in turn, increases the divergence between the elasticities in the two market segments. As noted in the previous Section, this increases the short-run revenue gain, k , and ultimately, long-run revenue and producer surplus gains. Such an increase in the short-run revenue gains, k , can be noted in Table 6.11.

Conversely, such promotion in the price elastic segment alone also reduces the price elasticity in this segment. However, in this case, this promotion strategy reduces the divergence between segment elasticities, and hence, reduces the short-run revenue gain, k . Once again, this can be noted in Table 6.11. In the long-run, this leads to lower increases in producer returns.

Caution must be exercised when considering the implications of these results, since the assumptions

underlying the models employed, which were noted in earlier parts of the study, must be borne in mind. However, they do illustrate that the influence of generic promotion differs in individual segments under discriminatory pricing without supply control, and it must not be presumed that such promotion is necessarily beneficial in all cases.

6.4.4. Conclusion

Long-run returns from market segmentation policies were analysed in this Section, and a number of observations made. In the first instance, long-run producer returns are lower when the price elasticity of demand in the aggregate market is high. Note that this composite elasticity is derived from individual segment elasticities, which are themselves influenced by a further set of factors in the long-run. These will be discussed in a subsequent Section. In a similar manner, long-run producer returns from market segmentation are likely to be lower in markets with price elastic supply. That is, if the supply response to price increases to a large degree over time, then the benefits to producers may diminish.

When the consequences of generic promotion are evaluated, it becomes obvious that its influence on producer returns depends on where this promotion has been targeted. Producer returns will be greater when such promotion is targeted towards the less price elastic segment, rather than the more price elastic segment.

6.5 FACTORS INFLUENCING SPECIFIC LONG-RUN VARIABLES

The factors which influence long-run variables such as the demand shift from advertising in an individual market segment, V_i , and the price elasticities of demand in these segments, n_i , were isolated previously and noted in Table 6.2. The former variable, V_i , is influenced by a number of factors which were discussed in the previous Chapter. Since these factors were not actually incorporated into the model, they will not be considered further. However, some of those variables which influence the individual demand elasticities, n_i , merit further attention. In particular, this applies to the competitive supply elasticities, e_{ii} , and the price transmission elasticities, ϕ_i .

Recall equation (6.3) where $|n_i|$ can be expressed as follows.

$$(6.3a) \quad |n_i| = \phi_i \left[|n_{ii}| \frac{C_i}{X_{i.a}} + e_{ii} \frac{S_{i.a}}{X_{i.a}} \right]$$

where ϕ_i represents all consumer and producer price transmission elasticities which are assumed equal,

$|n_{ii}|$ is the price elasticity of demand (absolute value) in market segment i ,

e_{ii} is the price elasticity of supply for competitors and is assumed to be equal for all supply sources,

$\frac{X_{i.a}}{C_i}$ is the market share held by the
by the marketing agency in country i,
and,

$\frac{S_{i.a}}{X_{i.a}}$ is the ratio of product supplied
from all other sources to that supplied
by the marketing agency.

Consider the change in the demand elasticity, $|n_i|$, with respect to a change in the competitive supply elasticities, e_{i1} . Differentiating (6.3a) with respect to e_{i1} gives

$$(6.6) \quad \frac{\delta |n_i|}{\delta e_{i1}} = \phi_i \frac{S_{i.a}}{X_{i.a}} > 0$$

Similarly, considering the impact of a change in the price transmission elasticities, ϕ_i , on $|n_i|$ yields

$$(6.7) \quad \frac{\delta |n_i|}{\delta \phi_i} = \left(|n_{i1}| \frac{C_i}{X_{i.a}} + e_{i1} \frac{S_{i.a}}{X_{i.a}} \right) > 0$$

That is, demand elasticities in individual market segments will rise when both competitive supply elasticities, e_{i1} , and price transmission elasticities, ϕ_i , also rise. The implications of these observations for marketing agencies will now be considered.

Competitive supply elasticities determine the magnitude of the competitive supply response. If, over time, such a response occurs in all market segments then competitive supply elasticities will increase. This, in turn, will cause all demand elasticities to increase. All else remaining equal, it has been shown in previous

Sections that this will reduce producer returns from market segmentation. Therefore, a marketing agency which practises market segmentation policies in a number of segments can expect a dissipation of producer returns from this activity in the long-run, with this dissipation being larger when competitive supply elasticities are high.

Price transmission elasticities capture, among other things, the extent to which an agency's product is differentiated from that of a competitor, with ϕ_i moving from unity towards zero as the degree of product-differentiation increases. A decrease in price transmission elasticities can be interpreted to mean that a change in a marketing agency's pricing behaviour will induce less of a response from competitors when reacting to such pricing strategies than would otherwise occur. Equation (6.7) implies that a reduction in price transmission elasticities will make the demand for an agency's product, n_i , less elastic than it would otherwise be.

Since an agency can influence the transmission elasticities, and hence the demand elasticities, facing it through product-differentiating promotion, the influence of changing price transmission elasticities in individual market segments on producer returns will now be investigated. Since it is the actual change in the magnitude of the demand elasticity, n_i , which influences the magnitude of producer returns, specified changes in these elasticities were postulated, and the required change in the price transmission elasticities to produce

such changes in demand elasticities noted. For the purposes of this exercise, it has been assumed that all price transmission elasticities, ϕ_i , were 0.5 in the base example.

The simulation results are presented in Table 6.12, where the base example is shown as the first observation. When product-differentiating promotion is undertaken in the relatively inelastic segment alone, as shown in the second simulation, the aggregate demand elasticity decreases, and the elasticity divergence between segments increases. As a result, the gains in short-run revenue, total revenue and producer surplus from market segmentation are far greater than those returned in the absence of product-differentiating promotion. Conversely, when product-differentiating promotion is undertaken in the price elastic segment alone, the divergence between elasticities is reduced. This results in producer returns from market segmentation being considerably lower than they would have been when no such promotion had occurred.

In order to investigate this result further, an additional set of simulations were conducted. In the fourth simulation product-differentiating promotion was undertaken in both segments. This resulted in considerable increases in short-run revenue, total revenue and producer surplus, although these were not as great as they would have been if product-differentiating promotion had been undertaken in segment 1 alone. A further promotion possibility was investigated in the final

ASSUMED EFFECT	PARAMETER CHANGES						CHANGES IN PRODUCER RETURNS		
	n_1	n_2	n_a	$(n_2 - n_1)$	ϕ_1	ϕ_2	$k(\$m)$	$\Delta TR(\$m)$	$\Delta PS(\$m)$
1. $\Delta n_1 = 0.0$ $\Delta n_2 = 0.0$	0.5	1.5	1.0	1.0	0.5	0.5	20.8	20.4	10.2
2. $\Delta n_1 = -0.25$ $\Delta n_2 = 0.0$	0.25	1.5	0.875	1.25	0.25	0.5	74.4	65.6	32.8
3. $\Delta n_1 = 0.0$ $\Delta n_2 = -0.25$	0.5	1.25	0.875	0.75	0.5	0.42	16.0	14.8	7.4
4. $\Delta n_1 = -0.25$ $\Delta n_2 = -0.25$	0.25	1.25	0.75	1.0	0.25	0.42	66.7	54.5	27.2
5. $\Delta n_1 = -0.25$ $\Delta n_2 = +0.25$	0.25	1.75	1.0	1.5	0.25	0.58	80.3	75.4	37.7
6. $\Delta n_1 = +0.25$ $\Delta n_2 = +0.25$	0.75	1.75	1.25	1.0	0.75	0.58	9.5	10.5	5.2

TABLE 6.12: THE INFLUENCE OF PRODUCT-DIFFERENTIATING PROMOTION UNDER DISCRIMINATORY PRICING

simulation, where product-differentiating promotion was undertaken in the inelastic segment, and promotion efforts in the elastic segment were aimed at increasing the perception of product homogeneity with respect to competitors' products. Such a policy increases the divergence between elasticities quite markedly, and of all the simulations considered, this promotion policy leads to the greatest increase in producer returns.

Obviously, caution must be exercised in interpreting the above results, since they merely show the influence of given changes in demand elasticities on producer returns, and give no indication of the promotion expenditure required to induce the various changes in price transmission elasticities which would be necessary to achieve these changes in demand elasticities.

However, the analysis does suggest that the presumption that product-differentiating promotion in all market segments is necessarily beneficial, is naive under conditions where optimal pricing is practised without supply control.

6.6 THE INFLUENCE OF MARKET SEGMENTATION POLICIES ON PRODUCER PRICE VARIABILITY

The relationship between market characteristics and market segmentation policies with respect to their influence on producer returns has been considered in some depth. However, the influence of such policies on the secondary producer objective of reducing price variability

is also important. Such an effect will now be considered. In the first instance, the change in producer price with respect to a change in output will be examined under both competitive and discriminatory pricing, and the relative price variability under both pricing regimes compared. In the second case, producer price variability under discriminatory pricing will be compared for alternative sets of market characteristics.

For a given output, Q , then the price which producers receive is given by

$$(4.2) \quad P_a = a_a - b_a Q$$

and the average revenue which they receive under discriminatory pricing can be represented by

$$(4.21) \quad P_{a,d} = a_a - b_a Q + \frac{k}{Q}$$

where $k > 0$

By differentiating equations (4.2) and (4.21) with respect to a change in output, Q , the change in producer price under each pricing regime can be determined. This gives

$$(6.8) \quad \left| \frac{\delta P_a}{\delta Q} \right| = b_a$$

and

$$(6.9) \quad \left| \frac{\delta P_{a,d}}{\delta Q} \right| = b_a + \frac{k}{Q^2}$$

Since $k > 0$ and $Q > 0$, then $\frac{k}{Q^2} > 0$, which implies that

$$(6.10) \quad \left| \frac{\delta P_{a,d}}{\delta Q} \right| > \left| \frac{\delta P_a}{\delta Q} \right|$$

That is, under the linear demand assumptions employed in each of these models, producer price will vary more with respect to a change in output under discriminatory pricing than it will under competitive pricing.

Consider discriminatory pricing under alternative sets of market characteristics. Specifically, consider two examples, one of which has relatively less price elastic demand at the competitive equilibrium, (P, Q) , and one of which has aggregate demand which is relatively more elastic at this point. In this case, average revenue under discriminatory pricing in the relatively less price elastic situation is given by

$$(6.11) \quad P_{ad1} = a_{a1} - b_{a1}Q + \frac{k_1}{Q}$$

Similarly, for the more price elastic case, this average revenue is given by

$$(6.12) \quad P_{ad2} = a_{a2} - b_{a2}Q + \frac{k_2}{Q}$$

Differentiating each of these expressions with respect to Q gives

$$(6.13) \quad \left| \frac{\delta P_{ad1}}{\delta Q} \right| = b_1 + \frac{k_1}{Q^2}$$

and

$$(6.14) \quad \left| \frac{\delta P_{ad2}}{\delta Q} \right| = b_2 + \frac{k_2}{Q^2}$$

By definition, $|n_{a1}| < |n_{a2}|$ at the competitive equilibrium (P, Q) , where n_{a1} refers to the aggregate price elasticity of demand in the appropriate case. This

implies that $b_1 > b_2$. If all else is assumed equal, then previous discussion in this Chapter implies that $k_1 > k_2$ where $|n_{a1}| < |n_{a2}|$.

Therefore,

$$(6.15) \quad \left| \frac{\delta P_{ad1}}{\delta Q} \right| > \left| \frac{\delta P_{ad2}}{\delta Q} \right|$$

That is, producer price variability will be higher when aggregate demand is less price elastic at the competitive equilibrium than it will be when such demand is more price elastic at this point.

However, previous analysis also suggests that where demand is relatively less price elastic at the competitive equilibrium, then returns from discriminatory pricing are relatively high. That is, when marketing agencies practise market segmentation policies, producer prices are likely to be more variable when the returns from these policies are high, and less variable when such returns are low. Therefore, factors which are conducive to favourable producer returns also lead to increased price variability when such policies are practised. The implication of this finding will be further discussed in the final Chapter.

6.7 AN APPLICATION TO THE NEW ZEALAND SHEEPMEATS INDUSTRY

In previous Sections, the influence of specific variables on market segmentation outcomes was investigated. This was done by varying the magnitude of each of these variables in turn with respect to a base numerical example. This approach yields valuable

information on the relationship between market characteristics and the outcomes of these policies. However, it gives no indication of what the actual returns from such policies might be in any particular industry. In this Section, therefore, the model which has been developed is applied to the New Zealand sheepmeats industry prior to the 1982 acquisition of industry output by the New Zealand Meat Producers' Board. This industry was selected for two reasons.

Firstly, the international sheepmeats industry is relatively competitive (Blyth, 1982), and hence competitive responses depicted in the model provide an accurate representation of behaviour in this industry. In addition, the Board was not actively engaged in market segmentation policies at that time, and therefore, the price and shares of output observed in individual market segments would provide a reasonable representation of a competitive allocation of output.

A second reason for investigating this industry is that estimates of some of the required elasticities are available from a recent model of the world sheepmeats market (Blyth, 1982). Unfortunately, competitive supply responses are absent, as are advertising responses. Although reasonable estimates of the competitive responses can be made, this is more difficult with advertising elasticities since very little is known about these responses in agricultural industries. Therefore, promotion aspects of the model will be excluded when returns from market segmentation are considered, although

comment will be made on the influence which promotion activity in alternative market segments is likely to have on producer returns.

The data used in this sheepmeat market analysis are presented in Table 6.13. New Zealand export data (New Zealand Meat Producers' Board, 1980), and world trade data (United States Department of Agriculture, 1982), were used to derive a set of trade flows between New Zealand and other exporters, and the major importing countries, for the 1979/80 season. Demand and price transmission elasticities were extracted from Blyth (1983). In the long-run, values of unity have been assumed for the own price elasticity of supply and cross price elasticities of supply between products from alternative sources.

In the empirical analysis, both short-run and long-run scenarios were simulated. The short-run situation will be considered first. In this case, it is assumed that the existing level of exports is reallocated among the eight market segments, and there is no response from other exporters. That is, the own and cross-price elasticities of supply are set to zero. Using the expression for measuring the export demand elasticities outlined previously, the short-run demand elasticities facing the New Zealand meat industry in individual market segments were derived by adjusting the market demand elasticities shown in Table 6.13 by the reciprocal of the market share held in that segment and the appropriate price transmission elasticity.

TABLE 6.13: DATA USED IN SHEEPMEAT MARKET ANALYSIS

=====

(a) Quantities ('000 tonnes)

Market	Production	Consumption	Imports	
			NZ	Total
Canada	5	19	10	14
USA	144	159	11	15
EEC (8)	350	410	13	102
UK	278	432	184	191
Iran	350	415	65	65
Japan	0	157	27	157
USSR	NA ¹	NA	62	157
Rest of World	NA	NA	78	165
Total			450	866

(b) Competitive Price

\$/tonne

All Markets

2,600

(c) Elasticities

Market	Demand	Price Transmission
Canada	-.99	1.0
USA	-.16	.81
EEC (8)	-.12	.94
UK	-.14	.94
Iran	-.28	.56
Japan	-.94	.64
USSR	-1.83	1.24
Rest of World	-.40	1.00

=====

1. NA - Not Available

Sources: New Zealand Meat Producers' Board
 USDA
 Blyth (1983)

These elasticities are shown in Table 6.14, along with the shares of output allocated to market segments which were observed at the 1979/80 price of \$2,600/tonne. This table also presents the optimal market shares and market prices under market segmentation. As would be expected, prices increase in markets with low demand elasticities, while the converse occurs in markets with high demand elasticities. The results suggest that under such a strategy, the average revenue would increase by 26 percent to \$3280 per tonne. This represents a gain to the industry of \$307 m., which in the absence of supply response would represent a direct gain in producer welfare.

The long-term gains from a segmentation strategy will now be considered. In this case, the assumed competitive supply response alters the demand elasticities facing the New Zealand industry. These modified elasticities are shown in Table 6.15. Similarly, the assumed own supply response increases industry output to 466,000 tonnes, which is then allocated among market segments according to optimal pricing procedures. The optimal prices, optimal market shares, and producer returns from this strategy are also shown in Table 6.15.

As a result of the competitive supply response, the aggregate demand elasticity, η_a , has increased markedly from its short-run value. In addition, the relative elasticities between markets have also changed, which affects the optimal market shares and relative prices. It is important to note that the range of optimal prices over

TABLE 6.14: MARKET SEGMENTATION IN THE SHEEPMET

MARKET - SHORT-RUN

Region	Elasticity (n_i)	Existing Market Share (s_i)	Optimal Market Share (s_{id})	Optimal Prices (P_{id})
Canada	1.88	.022	.023	2.53
USA	2.31	.024	.028	2.40
EC (8)	3.78	.030	.048	2.18
UK	0.32	.410	.243	5.90
Iran	1.78	.144	.147	2.57
Japan	5.46	.060	.126	2.07
USSR	4.63	.138	.250	2.12
ROW	0.85	.173	.129	3.36
Aggregate elasticity (n_a)	1.71			
Existing Price (\$'000/tonne)			2.6	
Optimal Average Revenue (\$'000/tonne)			3.28	
Quantity ('000/tonnes)			450	
Total Revenue (\$m)			1,170	
Optimal Total Revenue (\$m)			1,478	

TABLE 6.15: MARKET SEGMENTATION IN THE SHEEPMET MARKET -
LONG-RUN

Region	Regional Elasticity (n_i)	Optimal Prices (P_{id})	Existing Market Share (s_i)	Optimal Market Share (s_{id})
Canada	2.78	2.80	.022	.017
USA	11.81	2.42	.024	.050
EC (8)	37.48	2.37	.030	.124
UK	1.82	3.03	.410	.276
Iran	7.16	2.51	.144	.172
Japan	10.27	2.46	.060	.090
USSR	6.16	2.54	.138	.151
ROW	1.09	2.97	.173	.120
Aggregate elasticity (n_a)	5.14		1.00	1.00
Existing Price (\$'000/tonne)			2.6	
Quantity ('000 tonnes)			450	
Total Revenue (\$m)			1,170	
Optimal Average Revenue (\$'000/tonne)			2.69	
Optimal Quantity ('000 tonnes)			466	
Optimal Total Revenue (\$m)			1,253	
Change in Revenue (\$m)			83	
Change in Producer Surplus (\$m)			42	

the markets has lessened as the demands have become more elastic, and optimal average revenue is less than in the short-run case. The results show that in this particular environment, the average revenue has only increased by 3.5 percent, and total supply has increased by a similar amount. Thus the total revenue would be about seven percent higher than it is in the existing situation, and producer surplus would increase by only \$42 million.

A comparison between the short-run and long-run producer gains shown above indicates that while short-run producer returns from optimal pricing may appear favourable, in the long-run these returns are severely eroded by the competitive supply response to these policies. As a result, long-run producer surplus gains from market segmentation appear to be quite low in this industry. Obviously, these results are dependent on the assumptions made about the own and competitive supply response. However, they emphasize the importance of the competitive supply response in determining long-run outcomes of these policies.

As noted previously, promotion policies have not been incorporated into the model because of the absence of reliable estimates of advertising responses. However, comment can be made on the likely influence of advertising in alternative market segments. The regional demand elasticities in Table 6.15 suggest that a given shift in demand which results from generic advertising of New Zealand sheepmeats in a particular segment would have a more influential effect on producer surplus if it occurred

in the United Kingdom, Canada, or the residual Rest of World region. Product differentiating promotion which decreases demand elasticities in these segments would have a similar effect. However, the costs associated with both types of promotion activity, which would be made more obvious if advertising elasticities were known, would need to be offset against any increased returns from promotion in these segments.

CHAPTER 7

CONCLUSIONS

7.1 INTRODUCTION

This Chapter reviews the study, summarizes the results which were derived, and discusses their implications for marketing agencies. A summary of the research, including a list of the market conditions most appropriate for successful market segmentation is presented in Section 7.2. In Section 7.3, the results of the study are related to the literature, and directions for further research are indicated in Section 7.4. Finally, in Section 7.5, some implications of importance for marketing agencies are discussed.

7.2 SUMMARY

7.2.1 Outline of the Study

The general aim of this study was to investigate the nature of benefits to producers from market segmentation through controlled export marketing within the organizational context of a typically structured agricultural industry. Before doing this, the extent to which market segmentation policies were significant in agricultural exporting industries was investigated. On the basis of this, it was concluded that the major agricultural exporting industries were moving towards such policies.

In order to achieve the study objective, it was necessary to determine how marketing agencies operating in agricultural exporting industries are typically structured and what their objectives are. Once again, this was done through analysis of the major agricultural exporting industries. It was determined that a representative marketing agency would have the power to direct specified volumes of product to specific destinations or end uses, and to return a pooled price to producers. It would also have the necessary authority to acquire funds for promotion or product development, which may be necessary elements of a market segmentation policy. However, it would not have the power to control the level of industry output.

It was determined that agricultural marketing institutions often have three general objectives. These are to increase producer returns, to stabilize producer prices, and to ensure equity among producers. This final objective would be automatically fulfilled by the pool payment procedures adopted by a typically structured marketing agency. Of the other two objectives, increasing producer returns appears to be most important, with price stabilization seen as secondary.

The bulk of the study was devoted to constructing an analytical model which could be used to determine the magnitude of benefits from market segmentation, and which incorporated the appropriate organizational and market features. With the exception of promotion, the marketing mix components of a market segmentation policy are

encapsulated in the standard economic model of third-degree price discrimination. In an unmodified form, however, this model does not consider the producers' supply response to discriminatory pricing, nor does it include any competitive supply response which might result from such a pricing policy.

Consequently, the product allocation procedure which maximizes producer surplus where an agency does not have the power to control supply was determined. This condition was then incorporated into an algebraic model which calculated the returns to producers from optimal pricing in any number of market segments. This model took into consideration the appropriate own and competitive supply responses to this pricing policy.

Promotional aspects of the marketing mix were then included in the model. As a prerequisite to this, optimal promotion behaviour in alternative market segments under both competitive and discriminatory pricing was considered, and optimal rules for the allocation of promotion under these conditions were derived. While these results yielded insights into an appropriate allocation of promotional expenditure, they could not be directly incorporated into the model which calculated returns to producers from market segmentation activity. However, the returns from a given shift in demand or a given change in demand elasticities which result from either generic or product-differentiating promotion in alternative market segments were included in the model.

Expressions derived from the model were then used to evaluate the impact of market characteristics on the outcomes of market segmentation policies. In this way, it was possible to isolate those market conditions under which market segmentation activities by statutory marketing agencies are likely to be beneficial in terms of producer objectives. In order to observe what the actual benefits from market segmentation might be in a particular industry, the model was applied in a spatial dimension to the exports of New Zealand sheepmeats.

7.2.2 A Summary of Results

From the model, it was determined that market segmentation outcomes are influenced by a number of variables, where the magnitude of these variables is measured at their non-discriminating, competitive equilibrium values.

In the short-run, the most influential variables appear to be price elasticities of demand in individual market segments, and by implication, the demand elasticity in the aggregate market. It was determined that, in both absolute and relative terms, short-run returns will be large when, in competitive equilibrium, the aggregate demand elasticity is low and the divergence between demand elasticities in market segments is large. In absolute terms only, such returns will be high when the size of the industry before market segmentation is large, and the corresponding competitive price is high.

In addition to the above variables, producer returns in the long-run are influenced by an additional set of factors. In particular, such returns will be high when the price elasticity of supply is low. However, it must be noted that producer returns are much more sensitive to changes in the magnitude of the demand elasticity over time than they are to similar changes in the supply elasticity. Long-run producer returns will also be affected by demand shifts induced by generic promotion. In this case, producer returns from optimal pricing will be greater than those under competitive pricing when such promotion is undertaken in the relatively less price elastic segment. However, the converse applies when this type of promotion occurs in the relatively more price elastic segment. It would appear, therefore, that producer returns from optimal pricing will be relatively greater when generic promotion is undertaken in a less price elastic segment rather than a more price elastic one.

Since the simulations indicated that the magnitude of producer returns are sensitive to changes in the aggregate demand elasticity, factors which influence the long-run value of this variable were also summarized. Competitive supply elasticities are particularly important in this respect. As these elasticities rise over time, long-run demand elasticities also rise. Price transmission elasticities also influence long-run demand elasticities, with these falling as price transmission becomes less perfect through activities such as product-differentiating

promotion. If the net effect of the influence of such variables is to increase demand elasticities in all market segments, then as noted above, long-run producer returns will fall.

The magnitude of a demand shift in a market segment which results from generic advertising is also influenced by a further set of variables. Although these were not investigated in any depth, the observation can be made that any such demand shift will be greater when the proportionate increase in advertising expenditure is large and demand is relatively responsive to advertising. The influence of these resulting demand shifts on producer returns, however, depends on the segment in which such a shift has occurred.

The above discussion indicates the market conditions under which producer returns from market segmentation are likely to be high. In addition to increasing producer returns, however, marketing agencies will be interested in reducing producer price variability. Unfortunately, the study indicates that those market conditions which are conducive to high producer returns from market segmentation are also those which increase the variability in prices which producers receive. Hence, a conflict between satisfying the primary and secondary producer objectives arises when market segmentation policies are implemented.

7.2.3 Qualification of the Model and its Results

At this point, it is appropriate to qualify the conclusions reached in the study. The model which analysed market segmentation behaviour was constructed within a partial equilibrium comparative static framework. A partial equilibrium approach assumes that the market in question is separable from all other markets thereby implying that prices in these other markets are exogenous to the model. However, these ceteris paribus assumptions may not apply in some cases and care must be exercised in this respect when applying the model. In addition, the comparative static nature of the model abstracts from the adjustment path of variables towards their equilibrium values after market segmentation. As such, the model can cope with a short-run perspective, when policies have been implemented and no response to these policies has occurred, and a long-run perspective, where such responses are incorporated. However, it does not trace the dynamic change in producer returns which occurs between these two outcomes.

The model also assumes that the pricing response of competitors to market segmentation policies is essentially competitive. When any of the various oligopolistic pricing responses more typically represent market reactions to these policies, then the above model may not be the most appropriate analytical tool for evaluating market segmentation outcomes. The model structure also assumes that an industry is in competitive equilibrium before it introduces segmentation policies. In those cases

where such policies have been partially instituted, it may be necessary to make appropriate data transformations before the outcomes of optimal segmentation policies can be evaluated.

A further qualification to the use of the model arises from the linear functional form imposed on all demand and supply equations in the model. Therefore, caution must be exercised when interpreting model results if it is suspected that the true functional form of these equations is not linear, and that the market segmentation solution strays too far from the competitive solution, thereby invalidating the neighbourhood assumption used to justify the use of linear equations under non-linear conditions.

Finally, it must be emphasized that the returns from market segmentation calculated in the model represent gross returns from this activity, and do not incorporate the costs associated with actually segmenting a market or preventing arbitrage between segments. As such, they represent the maximum returns which can be expected from implementing these policies.

Notwithstanding the above caveats, the features of the model are robust enough for the purposes of the present study. In this case, the emphasis is on isolating the market conditions under which market segmentation activities by statutory marketing agencies operating in typically structured agricultural industries are likely to be beneficial in terms of producer objectives. As such, the focus is on more general factors, rather than on some of the detailed observations discussed above.

7.3 A REVIEW OF RESULTS WITH RESPECT TO THE LITERATURE

7.3.1 Introduction

The results which were derived from the model will now be reviewed with respect to the appropriate literature. Short-run market segmentation outcomes will be discussed initially, followed by an assessment of the conclusions reached on the own and competitive supply responses. Comment will then be made on aspects of promotion in relation to the literature. Finally, the findings on price variability will be examined.

7.3.2 Short-Run Considerations

The conditions for successful short-run market segmentation by a marketing agency noted in the study do not appear to have attracted a great deal of attention in the agricultural economics literature. However, a qualitative observation from the general economics literature is that a monopoly firm must face a downward sloping demand curve in aggregate, and individual demand curves must not be isolastic at the competitive price for price discrimination to be successfully practised (Scherer, 1980). Price discrimination by an agency which does not control the level of output is equivalent to this monopoly case in the short run, and the analysis in this study supports Scherer's observations. In addition, however, it generalizes and quantifies them.

7.3.3 Own Supply Response

Concern has been expressed in the literature that price discrimination among market segments will induce an own supply response which will reduce producer returns to their original level (Baritelle and Price, 1974; Weisenborn, 1969).

The analysis in this study suggests that where some inelasticity in supply remains, then long-run producer surplus gains will always be greater than what they would have been under a non-discriminating pricing policy. This conclusion supports a similar observation by Gardner (1983). Under these supply conditions also, the pool price returned to producers will always be greater than the competitive non-discriminating price. However, in the long-run, producer returns and the average pool price will be less than that realized in the short-run, an effect also noted by Reeves and Longmire (1982). Therefore, there is some support for the argument expounded by Wiesenborn (1969), that any supply response should be suppressed if maximizing producer returns is a primary objective. However, if such an objective is dominant, then a logical policy would be not only to suppress the supply response, but to restrict output further, from the competitive equilibrium output level back to the monopoly output level. In this way, producers capture additional gains from behaving as a price discriminating monopolist.

The second concern which dominates the literature with respect to an own supply response is that such a response leads to sub-optimal resource use, since the average

(pool) price which producers receive is greater than the marginal revenue in the most elastic market segment. Therefore, the cost of the additional supply is greater than the marginal revenue received for this effort (Banks and Mauldon, 1966; Freebairn and Gruen, 1977). Although the focus of this study is essentially parochial to the extent that market segmentation policies are being evaluated in terms of producer, rather than national objectives, this effect will nevertheless be commented on.

If it is assumed that optimality is meaningful in a national rather than a global (international) sense, then it would seem that where a marketing agency operates in a price-discriminating manner on export markets, and there is no linkage between the domestic price paid by consumers and the pool price returned to suppliers from activity in export markets, then allowing a supply response to occur is Pareto sub-optimal to the extent that returns to producers, and therefore to the nation, are not maximized. In these circumstances, the optimal level of industry output for export would appear to be the monopoly profit-maximizing level. When a linkage exists between domestic consumer prices and the price received by suppliers for export returns, or where the domestic market is relatively price inelastic and included in the price discrimination scheme, then it must be determined if gains to producers, in the form of increased returns, exceed losses to domestic consumers, in the form of higher prices, before a judgment can be made as to whether this

price-discriminating behaviour improves or reduces national welfare.

7.3.4 Competitive Supply Response

A competitive supply response to market segmentation policies was incorporated through its influence on export demand elasticities. Although trade researchers have been concerned with the influence of various factors on the magnitude of export demand elasticities, much of this research has concentrated on price transmission elasticities (Bredahl, 1979; Johnson, 1977; Tweeten, 1977). With the exception of Powell (1959) who investigated the influence of market share on export demand elasticities, there appears to be little research on the relative long-run influence of other factors such as the responsiveness of competitors.

However, the sheepmeats example investigated in this study indicated that this factor leads to a significant increase in the magnitude of export demand elasticities, thereby leading to a reduction in producer returns from optimal pricing over time. This supports the observations of other researchers who have expressed the opinion that optimal allocation policies will be constrained by potential competition, and that such competition could invalidate such allocation policies (Banks and Mauldon, 1966; Weisenborn, 1969; Edwards, 1970; Freebairn and Gruen, 1977).

7.3.5 Promotion

A review of the literature on promotion revealed that there was little information on how advertising effort should be optimally allocated across market segments under either competitive or discriminatory pricing. Although de Boer (1977) considered how to direct advertising effort under two-price schemes, he assumed monopolistic supply features. Alternatively, Nerlove and Waugh (1961) incorporated the appropriate supply response, but their analysis was restricted to an aggregate market. By investigating optimal advertising under appropriate supply conditions for alternative pricing regimes, this study extends the Nerlove-Waugh model and removes the single market assumption for which it has been criticized (Strak and Gill, 1983).

The effect on producer returns of a shift in demand through generic promotion has been well recognized in the literature (May, 1977), but there appears to have been little investigation of how such promotion in individual market segments influences producer returns. Similarly, product-differentiating promotion is seen to be desirable for producers (Parish, 1963), since it allows an agency to extract greater monopoly rent from consumers. However, as with generic promotion, there appears to have been little investigation on how this product-differentiating promotion in individual market segments influences producer returns when pricing is discriminatory. In this study, a preliminary investigation of this problem was undertaken, with the results suggesting that promotion

effort should be carefully targeted to specific market segments when pricing between these segments is discriminatory.

7.3.6 The Influence on Producer Price Variability

It has long been assumed that price discrimination schemes stabilize producer prices, and instances of this belief have been documented by Myers and Piggott (1981). The theoretical reasoning behind this presumption is unclear, although Myers and Piggott (1981) cite the argument that the diversion of product onto a secondary market cushions changes in supply because of the relatively more elastic demand in this market. However, both Myers and Piggott (1981) and Alston and Freebairn (1986) question whether price discrimination schemes do stabilize producer prices under variable supply conditions.

The results of this study support the reservations of these authors on the stabilizing effects of such schemes. It was concluded that producer prices will be more variable when pricing across markets is discriminatory rather than competitive, with this variability being greater when returns to producers from such a policy are higher.

7.4 DIRECTIONS FOR FURTHER RESEARCH

The research conducted in this study could be extended in a number of directions, both theoretical and empirical. Consider some of these theoretical possibilities. This study assumed that market segments were predetermined

according to some spatial, temporal or product form characteristic. However, where a marketing agency has the ability to create further market segments, there may be scope for investigating producer returns from identifying additional segments and for determining the optimal number of segments.

There is an obvious need to extend the research on promotion which was explored in this study. In particular, further work on the relative magnitude of producer returns in a number of market segments under varying market conditions given both discriminatory and competitive pricing could be undertaken. It would be desirable for such research to incorporate more sophisticated linkages between advertising expenditure and demand shifts than the simple linear approximation suggested in this model. An adaptation of a multiplicative demand model, as used by Dewbre et al (1986), might be appropriate in this regard, since this would incorporate diminishing returns to advertising effort.

If it was suspected that a competitive supply response to discriminatory pricing behaviour did not characterize the market structure in a particular industry, then an attempt could be made to include various oligopolistic responses into the analysis. Similarly, non-competitive market behaviour characterized by trade barriers could be more explicitly considered.

Further extensions to the model could incorporate costs associated with segmenting a market and shifting

demand or altering demand elasticities in these segments. Obviously, alternative functional forms for demand and supply equations could be experimented with, although the algebra associated with this would be quite cumbersome, and simulation rather than analytical techniques may be more appropriate in this case. Finally, attempts could be made to track the dynamic adjustments by the industry to market segmentation behaviour by a marketing agency.

Despite the obvious manner in which the model could be extended in its theoretical dimension, a more urgent priority is to operationalize the model for specific industry cases. This was done for the sheepmeats industry, where many of the required responses could be extracted from an international trade model (Blyth, 1983). This highlighted the complex nature of some of the data requirements. In particular, cross-price elasticities of supply from alternative production sources were shown to have a significant influence on producer returns. Unfortunately, at this stage there are few theoretical models available for estimating such supply responses, and none exist for sheepmeats. One interesting exception though is a recent beef industry model developed by Goddard (1983). In that study estimates of the substitution and cross-price elasticities between products from alternative sources are derived from a modified market share model.

Similarly, information on advertising elasticities of demand is crucial to any extension of the analysis. However, there have been very few attempts to estimate

these responses (Dewbre et al, 1986; Quilkey et al, 1986) or to investigate the extent to which these responses are inversely related to price elasticities of demand, as a priori reasoning suggests (de Boer, 1977; Parish, 1963).

An obvious need exists, therefore, for estimates of these supply and advertising responses. Such information would have a number of uses beyond its input to the market segmentation model, and research in this direction would appear to be highly appropriate. This would allow the model to be more fully operationalized, thereby giving more precise estimates of the extent to which market segmentation policies benefit producers in particular industries.

7.5 IMPLICATIONS FOR MARKETING AGENCIES

At the outset of this study, concern was expressed as to whether the wholesale application of marketing management strategies was appropriate in agricultural industries. These reservations were based on the fact that the organizational and market features in these industries may differ from those of their non-agricultural counterparts, for which many of these marketing prescriptions were derived.

The results of the study suggest that any presumption that market segmentation techniques will be automatically successful in agricultural industries should be treated with caution, particularly in the longer term. For example, the analysis of market segmentation in New Zealand's sheepmeats industry showed that, in the

short-run, average revenue to producers increased by 26 percent. However, in the long-run, this average revenue increase was only 3.5 percent.

It was further concluded from the study that untargeted promotion is not necessarily beneficial when optimal pricing policies are being practised. In fact, the study indicated that poorly targeted promotion activity could actually reduce producer returns from market segmentation activity. The implication of this finding for marketing agencies is that indiscriminate promotion activity is inadvisable.

Agencies which implement such policies must be prepared to accept that there will be a trade-off between producer objectives. When such strategies increase producer returns, they may also increase producer price variability when supply is variable.

The above reservations do not imply that market segmentation activity by agricultural marketing agencies is unwarranted. They do suggest, however, that the success of such policies may vary from industry to industry, and that performance in one industry is not necessarily indicative of performance in another industry. Therefore, based on the model results, factors which marketing agencies should take into account when considering market segmentation policies will now be outlined.

Where an agency has access to an appropriate data base, such as an international trade model, it will be able to derive relatively precise estimates of producer

returns from segmentation activity. To implement the optimal pricing component of the model developed in this study, data would be required on the quantity of the product exported, both in aggregate and individual market segments, and the price received per unit of output. Details of trade flows from competitors to each market segment would also be required, thereby giving estimates of the agency's market share in each segment.

In addition to these trade flows, an agency would require estimates of various responses. These include the own price elasticity of supply, and the price elasticities of demand in individual market segments. In order to determine these export demand elasticities, information would be required for each segment on the market demand elasticity in the segment itself, competitive supply elasticities to the segment, and price transmission elasticities relevant for the segment.

To operationalize the promotion component of a segmentation policy, information for each segment would be required on the initial promotion level, any proposed increase in promotion expenditure, and the advertising elasticity of demand. An agency with access to this type of data base could then make a reasonably accurate quantitative assessment of outcomes to producers from market segmentation activity.

In many industries, however, such detailed data will not be available. In this case, an agency wishing to assess market segmentation opportunities will have to rely on a more qualitative assessment. Data which are likely

to be accessible would include information on product sources and trade flows. If this indicates that output from the agency's suppliers is small, both in absolute terms and relative to that of competitors, then it is probable that export demand elasticities facing the industry in each segment will be large. This implies that returns from market segmentation are likely to be low and that such a policy might not be viable.

If trade flow data indicates that the industry has a relatively important influence in export markets, then the next step would be to make a qualitative assessment of the relevant responses. Consider the market demand elasticities in each segment. If it seems likely that the demand for the product responds positively to changes in income, and has many substitutes or a few very close substitutes, then market demand elasticities in each segment may be quite high, implying low producer returns.

Supply elasticities could be likewise assessed. If the product in question has a relatively short production cycle and is carried out using inputs which have obvious alternative uses, then these elasticities are likely to be highly responsive in the long-run. Once again, this implies low producer returns from market segmentation activity.

The final set of responses which must be considered are the price transmission elasticities. If the agency's product is not strongly differentiated from that of competitors, then the erosion of producer gains from market segmentation in the long-run will be more rapid.

Natural product-differentiating factors include variation from the competitors' product in form or seasonal availability. Perceptions of product differentiation may also result from a strong brand image.

If a qualitative analysis of the above type indicates that market segmentation activity might be successful, then a marketing agency could increase the precision of its judgments by investing research resources to estimate the appropriate responses in a more precise manner. However, it would be equally important for it to consider how it might influence the outcomes of segmentation policies when these are implemented.

An obvious response might be to recommend restrictions on output by its own producers thereby suppressing the own supply response to this type of marketing activity. However, the sheepmeats example indicated that much of the reduction in producer surplus over time resulted from a competitive supply response to segmentation policies, rather than to an own supply response. Therefore, production quotas may do little to slow down the erosion of producer gains.

Where competitive supply elasticities are high a more appropriate response might be to alter the purchaser's perception of the agency's product in those market segments where demand is less price elastic. This product-differentiating behaviour could nullify, to some extent, the damaging effect of these competitive supply responses.

A more fundamental principle which an agency might consider is its segmentation base. The analysis in this study indicated that producer returns will be high when the divergence between demand elasticities in individual segments is large. Therefore, a producer agency would be advised to search for that division of its product which maximizes such a divergence. Possibilities include segmentation in space, form, time and perception, or combinations of these. For example, the sheepmeats analysis indicated that returns from spatial segmentation were minimal in the long-run.

As a consequence, where demand is known to have some price inelasticity in particular segments, then an appropriate strategy may be to enter a highly price elastic segment, into which product from less elastic segments can be diverted. This could encompass a processing option or any other market segment which takes undifferentiated product and where a marketing agency has minimal price influence. The agency could then concentrate on product differentiating activity in its less price elastic segment, thereby allowing it to further exploit this market.

It is of interest to note that marketing agencies appear to have been practising these types of product diversion strategies. For example, the New Zealand Meat Producers' Board recently withdrew product from the carcass segment of the trade by rendering down mutton. Similarly, the New Zealand Dairy Board is advocating the conversion of surplus butter to butteroil. However, such

policies tend to reflect short-term problems associated with stockpiles of product, whereas the results of this study suggest that long-term co-ordinated market segmentation strategies of this type could improve producer returns.

REFERENCES

- Abel, M. E. Price Discrimination in the World Trade of Agricultural Commodities. Journal of Farm Economics, 48 (2) : 194-208. 1966.
- Alston, J. M. and Freebairn, J. W. Producer Price Equalization. Contributed paper presented to the 30th Annual Conference of the Australian Agricultural Economics Society, Canberra, 3-5 February, 1986.
- Apple and Pear Marketing Act 1971. Wellington, Government Printer, 1971. 28p.
- Balderstone, J. S. and others. Agricultural Policy Issues and Options for the 1980's. Working Group Report to the Minister for Primary Industry. Canberra, Australian Government Publishing Service, 1982. 167p.
- Banks, E. L. and Mauldon, R. G. Effects of Pricing Decisions of a Statutory Marketing Board. Australian Journal of Agricultural Economics, 10 (1) : 1-13. 1966.
- Baritelle, J. W. and Price, D. W. Supply Response and Marketing Strategies for Deciduous Crops. American Journal of Agricultural Economics, 56 (2) : 245-253. 1974.
- Battelle Columbus Laboratories. Final Report on the Marketing of New Zealand Wool to New Zealand Wool Board. Ohio, Battelle. 1971. 73p.

- Bateman, D. I. Agricultural Marketing : A Review of the Literature of Marketing Theory and of Selected Applications. Journal of Agricultural Economics, 27 : 171-226. 1976.
- Blandford, D. West African Export Marketing Boards. In Hoos, S. (ed.) Agricultural Marketing Boards - An International Perspective. Cambridge, Massachusetts, Ballinger Publishing Company, 1979. p. 121-149.
- Blyth, N. The World Sheepmeat Market : Implications for Policy. Unpublished PhD Thesis, University of Canterbury, Lincoln College. 1982.
- Blyth, N. The World Sheepmeat Market : An Econometric Model. Research Report No. 138. Agricultural Economics Research Unit, Lincoln College. 1983. 129p.
- Brash, D. T. The Case for Coordinated Export Marketing. Paper presented to the National Convention of the Export Institute of New Zealand, Rotorua, 8 August, 1985. 12p.
- Bredahl, M. E., Meyers, W. H. and Collins, K. J. The Elasticity of Foreign Demand for U. S. Agricultural Products : The Importance of the Price Transmission Elasticity. American Journal of Agricultural Economics, 61 (1) : 58-63. 1979.
- Breimyer, H. F. The Economics of Agricultural Marketing : A Survey. Review of Marketing and Agricultural Economics, 41 : 115-65. 1973.

- Campbell, K. The State Marketing Board - Relic or Prototype? Australian Journal of Agricultural Economics, 17(3) : 179-188. 1973.
- Chiang, A. C. Fundamental Methods of Mathematical Economics. 2nd ed. Tokyo, McGraw Hill Kogakusha, 1974. 784p.
- Claycamp, H. J. and Massy, W. F. A Theory of Market Segmentation. Journal of Marketing Research, 5 : 388-94. 1968.
- Cassady, R. Techniques and Purposes of Price Discrimination. Journal of Marketing, 11 : 135-150. 1946.
- Chudleigh, P. D. New Zealand Wool : Towards an Improved Marketing System. Agricultural Administration, 5 : 31-43. 1978.
- Cohen, R. L. Further Reflections on Agricultural Marketing. Journal of Agricultural Economics, 14(4) : 428-437. 1961.
- Cronin, M. R. Export Demand Elasticities with Less than Perfect Markets. Australian Journal of Agricultural Economics, 23(1) : 69-72. 1979.
- Currie, J. M. and Hoos, S. Marketing Boards : A Comparative Summary. In Hoos, S. (ed.) Agricultural Marketing Boards - An International Perspective. Cambridge, Massachusetts, Ballinger Publishing Company, 1979. p. 287-298.
- Currie, J. M., Murphy, J. A. and Schmitz, A. The Concept of Economic Surplus and Its Use in Economic Analysis. Economic Journal, 81 : 741-799. 1971.

- Dairy Board Act 1961. Wellington, Government Printer, 1977. 56p.
- Davies, J. L. Reflections on the Marketing of Our Farm Produce. Journal of Agricultural Economics, 14(2) : 128-142. 1960.
- De Alessi, L. Property Rights, Transaction Costs, and X-Efficiency : An Essay in Economic Theory. American Economic Review, 73 : 64-81. 1983.
- De Boer, A. J. Rural Product Promotion : Economic Aspects of Promotability, Organisation and Public Assistance. Review of Marketing and Agricultural Economics, 45(4) : 121-145. 1977.
- Department of Statistics. New Zealand Official Yearbook. Wellington, Government Printer, 1984. 1054 p.
- De Vos, G. Agricultural Marketing Boards in the Netherlands. In Hoos, S. (ed.) Agricultural Marketing Boards - An International Perspective. Cambridge, Massachusetts, Ballinger Publishing Company, 1979. p. 239-267.
- Dewbre, J., Thomson, M. and Richardson, R. Responses to Wool Promotion in the United States : Some Preliminary Results. Contributed paper presented to the 30th Annual Conference of the Australian Agricultural Economics Society, Canberra, 3-5 February, 1986.
- Dorfman, R. and Steiner, P.O. Optimal Advertising and Optimal Quality. American Economic Review, 44(2) : 826-836. 1954.

- Edwards, D. R. An Econometric Study of the North American Lamb Market. Technical Paper No. 10. Agricultural Economics Research Unit, Lincoln College. 1970. 255p.
- Edwards, G. W. Some Considerations in Allocating Resources between Shifting Supply and Shifting Demand. Paper presented to the 28th Annual Conference of the Australian Agricultural Economics Society, Sydney, 7-9 February, 1984.
- Edwards, G. W. and Freebairn, J. W. The Gains from Research into Tradable Commodities. American Journal of Agricultural Economics, 66(1) : 41-49, 1984.
- Enke, S. Some Notes on Price Discrimination. Canadian Journal of Economics and Political Science, 30 : 95-109. 1964.
- Freebairn, J. W. and Gruen, F. H. Marketing Australian Beef and Export Diversification Schemes. Australian Journal of Agricultural Economics, 21(1) : 26-39. 1977.
- Gardner, B. L. Price Discrimination or Price Stabilization : Debating with Models of U. S. Dairy Policy. American Journal of Agricultural Economics, 65(2) : 763-768. 1983.
- Goddard, E. W. Models of the Beef Markets in Japan and South Korea. Occasional Paper No. 3. School of Agriculture, La Trobe University. 1983. 45p.

- Hoos, S. U. S. Marketing Agreements and Orders: A Retrospective View. In Hoos, S. (ed.) Agricultural Marketing Boards - An International Perspective. Cambridge, Massachusetts, Ballinger Publishing Company, 1979. p. 269-286.
- Johnson, P. R. The Elasticity of Foreign Demand for U. S. Agricultural Products. American Journal of Agricultural Economics, 59(4) : 735-736. 1977.
- Kiwifruit Marketing Licensing Regulations 1977. Wellington, Government Printer, 1977. 17p.
- Kohls, R. L. and Downey, D. Marketing of Agricultural Products. 4th ed. New York, Macmillan, 1972. 432p.
- Kohls, R. L. and Uhl, J. N. Marketing of Agricultural Products. 5th ed. New York, Macmillan, 1980. 612p.
- Kotler, P. Principles of Marketing. New Jersey, Prentice-Hall, 1980. 684p.
- Kotler, P. Marketing Management. 5th ed. New Jersey, Prentice-Hall, 1984. 792p.
- Koutsoyiannis, A. Non-Price Decisions. The Firm in a Modern Context. London, Macmillan, 1982. 671p.
- Lambin, J. J. Advertising, Competition and Market Conduct in Oligopoly over Time. Amsterdam, North Holland, 1976. 312p.
- Le Vay, C. Agricultural Co-operative Theory : A Review. Journal of Agricultural Economics, 34(1) : 1-44. 1983.
- Lindner, R. K. and Jarrett, F. G. Supply Shifts and the Size of Research Benefits. American Journal of Agricultural Economics, 60(1) : 48-58, 1978.

- Machlup, F. Characteristics and Types of Price Discrimination. In National Bureau of Economic Research. Business Concentration and Price Policy : A Conference of the Universities - National Bureau Committee for Economic Research. New York, Arno Press. 1975. 511p.
- Martin, S. K., Young, L. and Zwart, A. C. Optimal Pricing and Promotion for Agricultural Marketing Agencies. Research Report No. 177. Agricultural Economics Research Unit, Lincoln College. 1986. 21p.
- May, M. E. Generic Advertising. Oxford Agrarian Studies, 6 : 135-156. 1977.
- McCarthy, E. J. Basic Marketing : A Managerial Approach. 6th ed. Homewood, Illinois, Richard D. Irwin, 1978. (1st ed. 1960). 767p.
- McClelland, E. L., Polopolus, L. and Myers, L. H. Optimal Allocation of Generic Advertising Budgets. American Journal of Agricultural Economics, 53(4) : 565-572. 1971.
- Meat Export Control Act 1921-22. Wellington, Government Printer, 1980. 32p.
- Meat Industry Task Force. Report to the Minister of Agriculture. Wellington, 1983. 63p.
- Melamed, A. The Citrus Marketing Board of Israel. In Hoos, S. (ed.) Agricultural Marketing Boards - An International Perspective. Cambridge, Massachusetts, Ballinger Publishing Company, 1979. p. 179-208.
- Mishan, E. J. What is Producer's Surplus? American Economic Review, 58 : 1269-1282. 1968.

- Mishan, E. J. Cost-Benefit Analysis. 3rd ed. London, George Allen and Unwin, 1982. 447p.
- Morley, J. A. E. Marketing Boards. In Warley, T. K. (ed.) Agricultural Producers and their Markets. Oxford, Basil Blackwell, 1967. p. 341-351.
- Myers, R. and Piggot, R. The Stability Effects of Two-Price Schemes : Preliminary Analysis. Contributed paper presented to the 25th Jubilee Conference of the Australian Agricultural Economics Society, Christchurch, February, 1981.
- Nagle, T. Economic Foundations of Pricing. Journal of Business, 57(1, pt. 2) : s3-s26. 1984.
- Nerlove, M. and Waugh, F. V. Advertising without Supply Control : Some Implications of a Study of the Advertising of Oranges. Journal of Farm Economics, 43(4) : 813-837. 1961.
- New Zealand Apple and Pear Marketing Board. Annual Reports. 1980/81, 1981/82, 1982/83, 1983/84.
- New Zealand Dairy Board. Annual Reports. 1981/82, 1982/83, 1983/84.
- New Zealand Dairy Board. Dairy Exporter, 60(10) : 37. 1985.
- New Zealand Fruit Export Control Board. Annual Reports. 1928, 1931, 1933, 1935, 1938.
- New Zealand Kiwifruit Authority. Annual Reports. 1977/78, 1978/79, 1980/81, 1981/82, 1982/83, 1983/84.
- New Zealand Meat Producers' Board. Annual Reports. 1979/80, 1980/81, 1981/82, 1982/83, 1983/84.

- New Zealand Meat Producers' Board. The New Zealand Meat Producer, 13(2) : 6-9, 1985a.
- New Zealand Meat Producers' Board. The New Zealand Meat Producer, 13(3) : 7-12, 1985b.
- New Zealand Treasury. Economic Management. Wellington, Office of Minister of Finance. 1984. 325p.
- New Zealand Wool Board. Annual Reports. 1980/81, 1981/82, 1982/83, 1983/84.
- Parish, R. M. Possibilities for Promoting Farm Products. Australian Journal of Agricultural Economics, 7(1) : 27-34. 1963.
- Philips, L. The Economics of Price Discrimination. Cambridge, Cambridge University Press, 1983. 284p.
- Piggott, R. R. Potential Gains from Controlling Distribution of the United States Apple Crop. Search Agriculture, 6(2) : 1-21. 1976.
- Piggott, R. R. Agricultural Selling Cartels : Relative Co-operator and Non Co-operator Gains. Australian Journal of Agricultural Economics, 25(1) : 14-29. 1981.
- Pigou, A. C. The Economics of Welfare. London, Macmillan, 1920. 976p.
- Powell, A. Export Receipts and Extension in the Wool Industry. Australian Journal of Agricultural Economics, 3(2) : 64-74, 1959.
- Primary Products Marketing Act 1953. Wellington, Government Printer, 1979. 11p.

- Quilkey, J. J. Gunawardana, P. J. and McGrath, B.
Approximation of Optimal Promotion Allocation among
Markets - A Note. Contributed paper presented to the
30th Annual Conference of the Australian Agricultural
Economics Society, Canberra, 3-5 February, 1986.
- Rae, A. N. An Evaluation of a New Zealand Marketing
Board's Supply Diversion Strategies. Australian
Journal of Agricultural Economics, 22(1) : 1-21.
1978.
- Rae, A. N. The Role and Performance of Statutory
Marketing Organisations. Discussion Paper 56.
Department of Agricultural Economics and Farm
Management, Massey University. 1980. 13p.
- Reeves, G. W. and Longmire, J. L. A Note on the Theory
of Price Determination in the Australian Beef Market.
Review of Marketing and Agricultural Economics,
50(1) : 119-125. 1982.
- Robinson, J. The Economics of Impefect Competition.
London, Macmillan, 1933. 352p.
- Rose, R. N. Supply Shifts and Research Benefits :
Comment. American Journal of Agricultural Economics,
62(4) : 834-837, 1980.
- Samuelson, P. A. Foundations of Economic Analysis.
Cambridge, Harvard University Press. 1948. 447p.
- Scherer, F. M. Industrial Market Structure and Economic
Performance. 2nd ed. Chicago, Rand McNally, 1980.
632p.
- Schmalensee, R. The Economics of Advertising. Amsterdam,
North Holland, 1972. 312p.

- Schmalensee, R. Output and Welfare Implications of Monopolistic Third-Degree Price Discrimination. American Economic Review, 71 : 242-7, 1981.
- Schmitz, A. and McCalla, A. The Canadian Wheat Board. In Hoos, S. (ed.) Agricultural Marketing Boards - An International Perspective. Cambridge, Massachusetts, Ballinger Publishing Company, 1979. p. 79-99.
- Schultz, R. L. and Wittink, D. R. The Measurement of Industry Advertising Effects. Journal of Marketing Research, 13 : 71-75, 1976.
- Shepherd, A. R. Economic Rent and the Industry Supply Curve. Southern Economic Journal, 37 : 209-211. 1970.
- Sieper, E. Rationalising Rustic Regulation. CIS Research Studies in Government Regulation 2. St. Leonards, N.S.W. The Centre for Independent Studies, 1982. 87p.
- Simon, J. L. and Arndt, J. The Shape of the Advertising Function. Journal of Advertising Research 20 : 11-28. 1980.
- Strak, J. Optimal Advertising Decisions for Farmers and Food Processors. Journal of Agricultural Economics, 34(3) : 303-315. 1983.
- Strak, J. and Gill, L. An Economic and Statistical Analysis of Advertising in the Market for Milk and Dairy Products in the U.K. Bulletin No.189. Department of Agricultural Economics, University of Manchester. 1983. 152p.

- Thompson, S. R. and Eiler, D. A. Determinants of Milk Advertising Effectiveness. American Journal of Agricultural Economics, 59(2) : 330-335. 1977.
- Tisdell, C. The Promotion of Wool and Synthetic Fibre Blends : Some Alternative Strategies for the Wool Industry. Review of Marketing and Agricultural Economics, 44(3) : 101-113. 1976.
- Tomek, W. G. and Robinson, K. L. Agricultural Product Prices. 2nd ed. Ithaca, Cornell University Press, 1981. 367p.
- Tweeten, L. The Demand for United States Farm Output. Food Research Institute Studies, 7(3) : 343-69. 1967.
- Tweeten, L. The Elasticity of Foreign Demand for U. S. Agricultural Products : Comment. American Journal of Agricultural Economics, 59(4) : 737-738. 1977.
- United States Department of Agriculture. Foreign Agricultural Circular : Livestock and Poultry, Foreign Agricultural Service, Washington. 1982.
- Veeman, M. M. D. Marketing Boards in New Zealand: An Economic Analysis and Appraisal. (Unpublished PhD Thesis). University of California, Berkeley, U. S. A., 1972. 500p.
- Veeman, M. M. D. New Zealand Marketing Boards. In Hoos, S. (ed.) Agricultural Marketing Boards - An International Perspective. Cambridge, Massachusetts, Ballinger Publishing Company, 1979. p. 101-119.
- Wallace, T. D. Measures of Social Costs and Agricultural Programs. Journal of Farm Economics, 44(2) : 580-94. 1962.

- Warley, T. K. A Synoptic View of Agricultural Marketing Organisations in the United Kingdom. In Warley, T. K. (ed.) Agricultural Producers and their Markets, Oxford, Basil Blackwell, 1967. p. 328-340.
- Watson, A. S. Marketing Policy in Relation to Agricultural Development. In Proceedings of the Eighteenth International Conference of Agricultural Economists. Aldershot, Gower, 1983. p. 306-314.
- Waugh, F. V. Needed Research on the Effectiveness of Farm Products Promotions. Journal of Farm Economics, 41(2) : 364-376. 1959.
- Weisenborn, D. E. Allocation of Florida Orange Production among Alternative Product Forms and Market Sectors. American Journal of Agricultural Economics, 51(5) : 1134-1137. 1969.
- Woods, L. D. Historical Survey of Marketing of New Zealand Dairy Products, Wool, Meat and Pipfruit. Unpublished notes. Department of Agricultural Economics and Marketing, Lincoln College, 1981.
- Wool Industry Act 1977. Wellington, Government Printer, 1978. 48p.
- Wool Marketing Study Group. Final Report Prepared for New Zealand Wool Board and New Zealand Wool Commission. Wellington, Bayleys Secretarial Services Ltd. 1967. 91p.
- Yerex, D. and Haines, W. The Kiwifruit Story. Masterton, Agricultural Publishing Associates, 1983. 96p.

APPENDIX 1PROOFS TO EQUATIONS IN CHAPTER 4¹A1. DERIVATION OF EQUATION (4.2) FROM EQUATION (4.1b)

$$(4.2) \quad P_a = a_a - b_a Q_a$$

where P_a is the competitive price

Q_a is the total quantity demanded

$$a_a = \frac{\sum_{i=1}^m \frac{a_i}{b_i}}{\sum_{i=1}^m \frac{1}{b_i}}$$

$$\text{and } b_a = \frac{1}{\sum_{i=1}^m \frac{1}{b_i}}$$

and

$$(4.1b) \quad P_i = a_i - b_i Q_i; \quad a_i, b_i > 0$$

where P_i and Q_i are price and quantity, respectively,

in market segment i .

Rearranging (4.1b) gives

$$(A1.1a) \quad -Q_i = \frac{P_i - a_i}{b_i}$$

Now,

$$P_i = P_a$$

1. All algebraic proofs presented in this Appendix have been numerically verified against a simple simulation model.

Therefore,

(A1.1b)

$$-Q_i = \frac{P_a - a_i}{b_i}$$

and

$$(A1.2a) \quad Q_a = \sum_{i=1}^m Q_i$$

Therefore,

$$(A1.2b) \quad -Q_a = \sum_{i=1}^m \left[\frac{P_a - a_i}{b_i} \right]$$

Let $m = 2$. Then

$$\begin{aligned} -Q_a &= \frac{P_a}{b_1} - \frac{a_1}{b_1} + \frac{P_a}{b_2} - \frac{a_2}{b_2} \\ &= \left[\frac{a_1}{b_1} + \frac{a_2}{b_2} \right] + \left[\frac{1}{b_1} + \frac{1}{b_2} \right] P_a \end{aligned}$$

In general terms,

$$-Q_a = \sum_{i=1}^m \frac{a_i}{b_i} + \sum_{i=1}^m \frac{1}{b_i} P_a$$

Therefore,

$$(4.2) \quad P_a = a_a - b_a Q_a$$

$$\text{where } a_a = \frac{\sum_{i=1}^m \frac{a_i}{b_i}}{\sum_{i=1}^m \frac{1}{b_i}}$$

$$\text{and } b_a = \frac{1}{\sum_{i=1}^m \frac{1}{b_i}}$$

A2. PROOFS TO EQUATIONS (4.5a) AND (4.5b)

Competitive equilibrium is determined when equations (4.2) and (4.3) are equated. This gives

$$a_s - b_s Q_0 = c + d Q_0$$

That is,

$$a_s - c = (b_s + d) Q_0$$

Therefore,

$$(4.5b) \quad Q_0 = \frac{a_s - c}{b_s + d}$$

By substituting (4.5b) into (4.2)

$$(4.5a) \quad P_0 = a_s - b_s \left[\frac{a_s - c}{b_s + d} \right]$$

A.3 PROOF OF EQUATION (4.11)

$$(4.11) \quad \lambda = a_s - 2b_s Q_s$$

Take equation (4.10) and solve for the vector $\{Q_1 \ Q_2 \ \lambda\}$

$$(4.10) \quad \begin{bmatrix} 2b_1 & 0 & 1 \\ 0 & 2b_2 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ Q_s \end{bmatrix}$$

From Cramer's Rule,

$$(A3.1) \quad \begin{bmatrix} Q_1 \\ Q_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} 2b_1 & 0 & 1 \\ 0 & 2b_2 & 1 \\ 1 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} a_1 \\ a_2 \\ Q_s \end{bmatrix}$$

if the 3 x 3 matrix is non-singular

Let the 3x3 matrix be called B

Now,

$$(A3.2a) \quad B^{-1} = \frac{1}{|B|} (\text{adj } B) \quad |B| \neq 0$$

Find the adjoint of B

Now,

$$\begin{aligned} \text{cof } B &= \begin{bmatrix} +|b_{11}| & -|b_{12}| & +|b_{13}| \\ -|b_{21}| & +|b_{22}| & -|b_{23}| \\ +|b_{31}| & -|b_{32}| & +|b_{33}| \end{bmatrix} \\ &= \begin{bmatrix} -1 & 1 & -2b_2 \\ 1 & -1 & -2b_1 \\ -2b_2 & -2b_1 & 4b_1b_2 \end{bmatrix} \end{aligned}$$

Therefore,

$$(A3.3) \quad \text{adj } B = \begin{bmatrix} -1 & 1 & -2b_2 \\ 1 & -1 & -2b_1 \\ -2b_2 & -2b_1 & 4b_1b_2 \end{bmatrix}$$

Find the determinant of B

$$|B| = 2b_1(-1) + (1) - 2b_2$$

Therefore,

$$(A3.4) \quad |B| = -2(b_1 + b_2)$$

Therefore, substituting (A3.3) and (A3.4) into (A3.2a) gives

$$(A3.2b) \quad B^{-1} = \frac{1}{-2(b_1 + b_2)} \begin{bmatrix} -1 & 1 & -2b_2 \\ 1 & -1 & -2b_1 \\ -2b_2 & -2b_1 & 4b_1b_2 \end{bmatrix}$$

Substituting (A3.2b) into (A3.1) gives

$$\lambda = \frac{1}{-2(b_1 + b_2)} (-2b_2 a_1 - 2b_1 a_2 + 4b_1 b_2 Q_a)$$

That is,

$$(A3.5a) \quad \lambda = \frac{a_1 b_2 + a_2 b_1}{b_1 + b_2} - 2 \frac{b_1 b_2}{b_1 + b_2} Q_a$$

$$\text{Take} \quad \frac{b_1 b_2}{b_1 + b_2}$$

$$\begin{aligned} \text{Now} \quad \frac{b_1 b_2}{b_1 + b_2} &= \frac{1}{\left[\frac{b_1 + b_2}{b_1 b_2} \right]} \\ &= \frac{1}{\frac{b_1}{b_1 b_2} + \frac{b_2}{b_1 b_2}} \end{aligned}$$

$$\frac{b_1 b_2}{b_1 + b_2} = \frac{1}{\sum_{i=1}^2 \frac{1}{b_i}}$$

Therefore, from equation (4.2)

$$(A3.6) \quad \frac{b_1 b_2}{b_1 + b_2} = b_a$$

$$\text{Take} \quad \frac{a_1 b_2 + a_2 b_1}{b_1 + b_2}$$

$$\text{Now} \quad \frac{a_1 b_2 + a_2 b_1}{b_1 + b_2} = a_1 \frac{b_2}{b_1 + b_2} + a_2 \frac{b_1}{b_1 + b_2}$$

$$\begin{aligned}
 &= \frac{b_1 b_2}{b_1 + b_2} \left[\frac{a_1}{b_1} + \frac{a_2}{b_2} \right] \\
 &= \frac{\sum_{i=1}^2 \frac{a_i}{b_i}}{\sum_{i=1}^2 \frac{1}{b_i}}
 \end{aligned}$$

Therefore,

$$(A3.7) \quad \frac{a_1 b_2 + a_2 b_1}{b_1 + b_2} = a_a$$

Therefore,

$$(A3.5b) \quad \lambda = a_a - 2b_a Q_a$$

A4. PROOF OF EQUATION (4.15)

$$(4.15) \quad Q_{1d} = \frac{2b_a Q_a + a_1 - a_a}{2b_1}$$

Now, when $MR_1 = MR_a$

$$a_1 - 2b_1 Q_1 = a_a - 2b_a Q_a$$

That is,

$$-2b_1 Q_1 = -2b_a Q_a + a_a - a_1$$

Therefore,

$$(4.15) \quad Q_{1d} = \frac{2b_a Q_a + a_1 - a_a}{2b_1}$$

A5. PROOF OF EQUATION (4.16)

$$(4.16) \quad P_{1d} = \frac{1}{2}(a_1 + a_a) - b_a Q_a$$

From equation (4.1b),

$$P_{1d} = a_1 - b_1 Q_{1d}$$

$$= a_i - b_i \left[\frac{2b_a Q_a + a_i - a_a}{2b_i} \right]$$

$$= a_i - b_a Q_a - \frac{a_i}{2} + \frac{a_a}{2}$$

Therefore,

$$(4.16) \quad P_{id} = \frac{1}{2}(a_i + a_a) - b_a Q_a$$

A6. PROOFS OF EQUATIONS (4.17a) TO (4.17d)

$$(4.17a) \quad a_i = P_a \left[1 + \frac{1}{|n_i|} \right]$$

and

$$(4.17b) \quad b_i = \frac{P_a}{Q_i} \cdot \frac{1}{|n_i|}$$

Now, from equation (4.1b),

$$(A6.1) \quad P_a = a_i - b_i Q_i$$

$$\text{Now } b_i = - \frac{\delta P_i}{\delta Q_i}$$

$$= - \left[\frac{\delta P_i}{Q_i} \cdot \frac{Q_i}{P_i} \right] \frac{P_i}{Q_i}$$

$$= \frac{1}{|n_i|} \cdot \frac{P_i}{Q_i}$$

Since $P_i = P_a$, then

$$(4.17b) \quad b_i = \frac{1}{|n_i|} \cdot \frac{P_a}{Q_i}$$

Substituting (4.17b) into (A6.1) gives

$$\begin{aligned} P_a &= a_i - \left[\frac{1}{|n_i|} \cdot \frac{P_a}{Q_i} \right] Q_i \\ &= a_i - \frac{P_a}{|n_i|} \end{aligned}$$

Therefore,

$$(4.17a) \quad a_i = P_a \left[1 + \frac{1}{|n_i|} \right]$$

Consider

$$(4.17c) \quad a_a = P_a \left[1 + \frac{1}{|n_a|} \right]$$

and,

$$(4.17d) \quad b_a = \frac{1}{|n_a|} \cdot \frac{P_a}{Q_a}$$

Equations (4.17c) and (4.17d) can be proved in the same way as (4.17a) and (4.17b) using equation (4.2), where

$$(4.2) \quad P_a = a_a - b_a Q_a$$

instead of equation (4.1b) above.

A7. PROOF OF EQUATION (4.18)

$$(4.18) \quad s_{id} = \frac{1}{2} s_i \left[\frac{n_i}{n_a} + 1 \right]$$

Recall equation (4.15)

$$(4.15) \quad Q_{id} = \frac{2b_a Q_a + a_i - a_a}{2b_i}$$

Substitute equations (4.17a) to (4.17d) into (4.15)

Then,

$$Q_{id} = \frac{2 \cdot \frac{1}{n_a} \cdot \frac{P_a}{Q_a} \cdot Q_a + P_a + \frac{P_a}{n_i} - P_a - \frac{P_a}{n_a}}{2 \cdot \frac{1}{n_i} \cdot \frac{P_a}{Q_i}}$$

That is,

$$(A7.1) \quad Q_{id} = \frac{n_i Q_i}{n_a} + \frac{n_i Q_i}{2} \left[\frac{1}{n_i} - \frac{1}{n_a} \right]$$

Now,

$$(A7.2) \quad s_{id} = \frac{Q_{id}}{Q_a}$$

Substitute (A7.1) into (A7.2)

Therefore,

$$\begin{aligned} s_{id} &= \frac{1}{Q_a} \left[\frac{n_i Q_i}{n_a} + \frac{n_i Q_i}{2} \left[\frac{1}{n_i} - \frac{1}{n_a} \right] \right] \\ &= \frac{n_i Q_i}{n_a Q_a} + \frac{n_i Q_i}{2 n_i Q_a} - \frac{n_i Q_i}{2 n_a Q_a} \end{aligned}$$

That is,

$$(4.18) \quad s_{id} = \frac{1}{2} s_i \left[\frac{n_i}{n_a} + 1 \right]$$

A8. PROOF OF EQUATION (4.19)

$$(4.19) \quad P_{id} = P_a + \frac{1}{2} P_a \left[\frac{1}{n_i} - \frac{1}{n_a} \right]$$

Recall equation (4.16)

$$(4.16) \quad P_{id} = \frac{1}{2} (a_i + a_a) - b_a Q_a$$

Substitute equations (4.17a), (4.17c) and (4.17d) into equation (4.16)

This gives

$$\begin{aligned}
 P_{1d} &= \frac{1}{2} \left[P_a + \frac{P_a}{n_i} + P_a + \frac{P_a}{n_a} \right] - \frac{1}{n_a} \cdot \frac{P_a}{Q_a} \cdot Q_a \\
 &= \frac{1}{2} \left[2P_a + P_a \left[\frac{1}{n_i} + \frac{1}{n_a} \right] \right] - \frac{1}{n_a} \cdot P_a \\
 &= P_a + \frac{P_a}{2} \cdot \frac{1}{n_i} + \frac{P_a}{2} \cdot \frac{1}{n_a} - \frac{P_a}{n_a} \\
 &= P_a + \frac{P_a}{2} \cdot \frac{1}{n_i} - \frac{P_a}{2} \cdot \frac{1}{n_a}
 \end{aligned}$$

Therefore,

$$(4.19) \quad P_{1d} = P_a + \frac{1}{2} P_a \left[\frac{1}{n_i} - \frac{1}{n_a} \right]$$

A9. PROOF OF EQUATION (4.21)

$$(4.21) \quad P_{ad} = a_a - b_a Q_a + \frac{k}{Q_a}$$

$$\text{where } k = \sum_{i=1}^n \left[\frac{(a_i - a_a)^2}{4b_i} \right] > 0$$

Let $n = 2$

From equation (4.20a)

$$(A9.1) \quad P_{ad} = \frac{P_{1d} Q_{1d} + P_{2d} Q_{2d}}{Q_{1d} + Q_{2d}}$$

That is,

$$P_{ad} = \frac{1}{Q_a} (P_{1d} Q_{1d} + P_{2d} Q_{2d})$$

Substitute equation (4.1b) for P_{1d}

Therefore,

$$\begin{aligned} P_{ad} &= \frac{1}{Q_a} \left[(a_1 - b_1 Q_{1d}) Q_{1d} + (a_2 - b_2 Q_{2d}) Q_{2d} \right] \\ &= \frac{1}{Q_a} \left[a_1 Q_{1d} - b_1 Q_{1d}^2 + a_2 Q_{2d} - b_2 Q_{2d}^2 \right] \end{aligned}$$

Substitute equation (4.15) for Q_{1d}

$$\begin{aligned} P_{ad} &= \frac{1}{Q_a} \left[a_1 \left[\frac{2b_a Q_a + a_1 - a_a}{2b_1} \right] - b_1 \left[\frac{2b_a Q_a + a_1 - a_a}{2b_1} \right]^2 \right. \\ &\quad \left. + a_2 \left[\frac{2b_a Q_a + a_2 - a_a}{2b_2} \right] - b_2 \left[\frac{2b_a Q_a + a_2 - a_a}{2b_2} \right]^2 \right] \\ &= \frac{1}{Q_a} \left[a_1 \left[\frac{2b_a Q_a + a_1 - a_a}{2b_1} \right] \right. \\ &\quad \left. - b_1 \left[\frac{4b_a^2 Q_a^2 + 4a_1 b_a Q_a - 4a_a b_a Q_a - 2a_1 a_a + a_1^2 + a_a^2}{4b_1^2} \right] \right. \\ &\quad \left. + a_2 \left[\frac{2b_a Q_a + a_2 - a_a}{2b_2} \right] \right. \\ &\quad \left. - b_2 \left[\frac{4b_a^2 Q_a^2 + 4a_2 b_a Q_a - 4a_a b_a Q_a - 2a_2 a_a + a_2^2 + a_a^2}{4b_2^2} \right] \right] \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{Q_a} \left[\frac{4a_1 b_a Q_a + 2a_1^2 - 2a_1 a_a - 4b_a^2 Q_a^2 - 4a_1 b_a Q_a + ctd}{4b_1} \right. \\
&\quad \left. \frac{4a_a b_a Q_a - 2a_1 a_a - a_1^2 - a_a^2}{4b_1} \right] + \\
&\quad \left[\frac{4a_2 b_a Q_a + 2a_2^2 - 2a_2 a_a - 4b_a^2 Q_a^2 - 4a_2 b_a Q_a + ctd}{4b_2} \right. \\
&\quad \left. \frac{4a_a b_a Q_a + 2a_2 a_a - a_2^2 - a_a^2}{4b_2} \right] \\
&= \frac{1}{Q_a} \left[\frac{a_1^2 - a_a^2 + 4a_a b_a Q_a - 4b_a^2 Q_a^2}{4b_1} \right. \\
&\quad \left. + \frac{a_2^2 - a_a^2 + 4a_a b_a Q_a - 4b_a^2 Q_a^2}{4b_2} \right] \\
&= \frac{1}{Q_a} \left[\left[\frac{a_1^2 - a_a^2}{4b_1} + \frac{a_2^2 - a_a^2}{4b_2} \right] + \left[\frac{4a_a b_a Q_a}{4b_1} + \frac{4a_a b_a Q_a}{4b_2} \right] \right. \\
&\quad \left. - \left[\frac{4b_a^2 Q_a^2}{4b_1} + \frac{4b_a^2 Q_a^2}{4b_2} \right] \right]
\end{aligned}$$

That is,

$$\begin{aligned}
(A9.2) \quad P_{ad} &= \frac{a_a b_a b_1 + a_a b_a b_2}{b_1 b_2} - \left[\frac{b_1 b_a^2 + b_2 b_a^2}{b_1 b_2} \right] Q_a \\
&\quad + \frac{1}{Q_a} \left[\frac{a_1^2 - a_a^2}{4b_1} + \frac{a_2^2 - a_a^2}{4b_2} \right]
\end{aligned}$$

Take term 1 of this expression

Now,

$$\frac{a_a b_a b_1 + a_a b_a b_2}{b_1 b_2} = \frac{a_a b_a (b_1 + b_2)}{b_1 b_2}$$

From equation (A3.6), this

$$\begin{aligned} &= \frac{a_a b_a}{b_a} \\ &= a_a \end{aligned}$$

Take term 2 of this expression

Now,

$$-\left[\frac{b_1 b_a^2 + b_2 b_a^2}{b_1 b_2} \right] Q_a = -\left[\frac{b_a^2 (b_1 + b_2)}{b_1 b_2} \right] Q_a$$

which, from equation (A3.6),

$$\begin{aligned} &= -\frac{b_a^2}{b_a} Q_a \\ &= -b_a Q_a \end{aligned}$$

Take term 3

$$\begin{aligned} &\frac{1}{Q_a} \left[\left[\frac{a_1^2 - a_a^2}{4b_1} \right] + \left[\frac{a_2^2 - a_a^2}{4b_2} \right] \right] \\ &= \left[\sum_{i=1}^2 \left[\frac{a_i^2 - a_a^2}{4b_i} \right] \right] \frac{1}{Q_a} \end{aligned}$$

It can be shown that term 3

$$= \left[\sum_{i=1}^2 \left[\frac{(a_i - a_a)^2}{4b_i} \right] \right] \frac{1}{Q_a}$$

$$\text{Now } \sum_{i=1}^2 \left[\frac{a_i^2 - a_s^2}{4b_i} \right] = \frac{a_1^2}{4b_1} - \frac{a_s^2}{4b_1} + \frac{a_2^2}{4b_2} - \frac{a_s^2}{4b_2}$$

and

$$\sum_{i=1}^2 \left[\frac{(a_i - a_s)^2}{4b_i} \right] = \frac{a_1^2}{4b_1} - \frac{2a_1 a_s}{4b_1} + \frac{a_s^2}{4b_1} + \frac{a_2^2}{4b_2} - \frac{2a_2 a_s}{4b_2} + \frac{a_s^2}{4b_2}$$

Hence, it is required to prove that

$$-\frac{a_s^2}{4b_1} - \frac{a_s^2}{4b_2} = -\frac{2a_1 a_s}{4b_1} + \frac{a_s^2}{4b_1} - \frac{2a_2 a_s}{4b_2} + \frac{a_s^2}{4b_2}$$

Take the RHS of this expression

$$\begin{aligned} & -\frac{2a_1 a_s}{4b_1} + \frac{a_s^2}{4b_1} - \frac{2a_2 a_s}{4b_2} + \frac{a_s^2}{4b_2} \\ = & -\frac{2a_1(a_1 b_2 + a_2 b_1)}{4b_1(b_1 + b_2)} - \frac{2a_2(a_1 b_2 + a_2 b_1)}{4b_2(b_1 + b_2)} \\ & + \frac{(a_1 b_2 + a_2 b_1)^2}{4b_1(b_1 + b_2)^2} + \frac{(a_1 b_2 + a_2 b_1)^2}{4b_2(b_1 + b_2)^2} \\ = & \frac{-2a_1^2 b_2 - 2a_1 a_2 b_1}{4b_1(b_1 + b_2)} + \frac{-2a_2^2 b_1 - 2a_1 a_2 b_2}{4b_2(b_1 + b_2)} \\ & + \frac{(a_1 b_2 + a_2 b_1)^2}{4b_1(b_1 + b_2)^2} + \frac{(a_1 b_2 + a_2 b_1)^2}{4b_2(b_1 + b_2)^2} \\ = & \frac{-2a_1^2 b_2^2 - 2a_1 a_2 b_1 b_2 - 2a_1 a_2 b_1 b_2 - 2a_2^2 b_1^2}{4b_1 b_2 (b_1 + b_2)} \\ & + \frac{b_2(a_1 b_2 + a_2 b_1)^2 + b_1(a_1 b_2 + a_2 b_1)^2}{4b_1 b_2 (b_1 + b_2)^2} \end{aligned}$$

$$\begin{aligned}
&= \frac{-2a_1^2 b_2^2 - 4a_1 a_2 b_1 b_2 - 2a_2^2 b_1^2}{4b_1 b_2 (b_1 + b_2)} \\
&\quad + \frac{(b_1 + b_2)(a_1 b_2 + a_2 b_1)^2}{4b_1 b_2 (b_1 + b_2)^2} \\
&= \frac{-2a_1^2 b_2^2 - 4a_1 a_2 b_1 b_2 - 2a_2^2 b_1^2 + a_1^2 b_2^2 + 2a_1 a_2 b_1 b_2 + a_2^2 b_1^2}{4b_1 b_2 (b_1 + b_2)} \\
&= \frac{-a_1^2 b_2^2 - 2a_1 a_2 b_1 b_2 + a_2^2 b_1^2}{4b_1 b_2 (b_1 + b_2)} \\
&= -\frac{a_a^2 (b_1 + b_2)^2}{4b_1 b_2 (b_1 + b_2)} \\
&= -\frac{a_a^2 (b_1 + b_2)}{4b_1 b_2} \\
&= -\frac{a_a^2 b_1}{4b_1 b_2} - \frac{a_a^2 b_2}{4b_1 b_2} \\
&= -\frac{a_a^2}{4b_1} - \frac{a_a^2}{4b_2}
\end{aligned}$$

= L. H. S.

Therefore, collecting together the three terms, and substituting back into (A9.2) gives

$$(4.21) \quad P_{ad} = a_a - b_a Q_a + \frac{k}{Q_a}$$

$$\text{where } k = \sum_{i=1}^2 \left[\frac{(a_i - a_a)^2}{4b_i} \right] > 0$$

A10. PROOF TO EQUATION (4.22)

$$(4.22) \quad k = \frac{1}{4} P_a Q_a \sum_{i=1}^n \left[n_i s_i \left[\frac{1}{n_i} - \frac{1}{n_a} \right]^2 \right]$$

From equation (4.21)

$$(A10.1) \quad k = \sum_{i=1}^n \left[\frac{(a_i - a_a)^2}{4 b_i} \right]$$

Substitute equations (4.17a), (4.17b) and (4.17c) into equation (A10.1)

Therefore,

$$k = \sum_{i=1}^n \left[\frac{\left[P_a + \frac{P_a}{n_i} - P_a - \frac{P_a}{n_a} \right]^2}{4 \cdot \frac{1}{n_i} \cdot \frac{P_a}{Q_i}} \right]$$

$$= \sum_{i=1}^n \left[\frac{\left[P_a \left[\frac{1}{n_i} - \frac{1}{n_a} \right] \right]^2}{4 \cdot P_a \cdot \frac{1}{n_i Q_i}} \right]$$

$$= \frac{1}{4} \sum_{i=1}^n \left[n_i Q_i P_a \left[\frac{1}{n_i} - \frac{1}{n_a} \right]^2 \right]$$

$$= \frac{1}{4} \sum_{i=1}^n \left[n_i \frac{Q_i}{Q_a} Q_a P_a \left[\frac{1}{n_i} - \frac{1}{n_a} \right]^2 \right]$$

Therefore,

$$(4.22) \quad k = \frac{1}{2} P_a Q_a \sum_{i=1}^n \left[n_i s_i \left[\frac{1}{n_i} - \frac{1}{n_a} \right]^2 \right]$$

A11. PROOFS TO EQUATIONS (4.23a) AND (4.24a)

$$(4.23a) \quad P_{d1} = a_a - \frac{b_a(a_a - c + X^{0.5})}{2(b_a + d)} + \frac{2k(b_a + d)}{a_a - c + X^{0.5}}$$

and

$$(4.24a) \quad Q_1 = \frac{a_a - c + X^{0.5}}{2(b_a + d)}$$

where $X = (a_a - c)^2 + 4k(b_a + d)$ in both cases.

To find Q_1

First, equate equations (4.21) and (4.3) to give

$$a_a - b_a Q_1 + \frac{k}{Q_1} = c + d Q_1$$

That is,

$$-b_a Q_1 - d Q_1 + \frac{k}{Q_1} = (c - a_a)$$

Therefore,

$$-(b_a + d) Q_1 + \frac{k}{Q_1} - (c - a_a) = 0$$

That is,

$$(b_a + d) Q_1 - \frac{k}{Q_1} - (a_a - c) = 0$$

Therefore,

$$(b_a + d) Q_1^2 - (a_a - c) Q_1 - k = 0$$

Recall that the solutions to a quadratic equation

$ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm (b^2 - 4ac)^{0.5}}{2a}$$

Therefore,

$$Q_1 = \frac{-[-(a_a - c)] \pm \{[-(a_a - c)^2 - [4(b_a + d)(-k)]]\}^{0.5}}{2(b_a + d)}$$

Ignoring the negative root, this gives

$$(4.24a) \quad Q_1 = \frac{a_a - c + X^{0.5}}{2(b_a + d)}$$

$$\text{where } X = (a_a - c)^2 + 4k(b_a + d)$$

Recall (4.21)

$$(4.21) \quad P_{a,d} = a_a - b_a Q_a + \frac{k}{Q_a}$$

Substitute (4.24a) into this to find P_1

Therefore,

$$(4.23a) \quad P_{d,1} = a_a - \frac{b_a(a_a - c + X^{0.5})}{2(b_a + d)} + \frac{2k(b_a + d)}{a_a - c + X^{0.5}}$$

A12. PROOF TO EQUATIONS (4.17e) AND (4.17f)

$$(4.17e) \quad c = P_0 \left[1 - \frac{1}{e} \right]$$

and

$$(4.17f) \quad d = \frac{P_0}{Q_0} \cdot \frac{1}{e}$$

Now, from equation (4.3)

$$(A12.1) \quad P_0 = c + dQ_0$$

Now,

$$(A12.2) \quad d = \frac{\delta P_0}{\delta Q_0}$$

$$= \left[\frac{\delta P_0}{\delta Q_0} \cdot \frac{Q_0}{P_0} \right] \cdot \frac{P_0}{Q_0}$$

Therefore,

$$(4.17f) \quad d = -\frac{1}{e} \cdot \frac{P_0}{Q_0}$$

Substituting (4.17f) into (A12.1) gives

$$P_0 = c + \frac{1}{e} \cdot \frac{P_0}{Q_0} \cdot Q_0$$

Therefore,

$$(4.17e) \quad c = P_0 \left[1 - \frac{1}{e} \right]$$

A13. PROOFS TO EQUATIONS (4.23b) AND (4.24b)

$$(4.23b) \quad P_{d1} = P_0 \left[1 + \frac{1}{2n_a} \right] - \frac{P_0 Z^{0.5}}{Q_0 n_a} + \frac{k}{\frac{1}{2}Q_0 + Z^{0.5}}$$

and

$$(4.24b) \quad Q_1 = \frac{Q_0}{2} + Z^{0.5}$$

$$\text{where } Z = \frac{Q_0^2}{4} + \frac{kQ_0}{P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right]} \quad \text{in both cases}$$

Recall equation (4.24a)

$$(4.24a) \quad Q_1 = \frac{a_a - c + X^{0.5}}{2(b_a + d)}$$

$$\text{where } X = (a_a - c)^2 + 4k(b_a + d)$$

Substitute equations (4.17c) to (4.17f) into (4.24a).

This gives

$$\begin{aligned}
 Q_1 &= \frac{\left[\frac{P_0}{n_a} + \frac{P_0}{e} \right] + \left[\left[\frac{P_0}{n_a} + \frac{P_0}{e} \right]^2 + 4k \left[\frac{1}{n_a} \cdot \frac{P_0}{Q_0} + \frac{1}{e} \cdot \frac{P_0}{Q_0} \right] \right]^{0.5}}{2 \left[\frac{1}{n_a} \cdot \frac{P_0}{Q_0} + \frac{1}{e} \cdot \frac{P_0}{Q_0} \right]} \\
 &= \frac{P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] + \left[\left[P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] \right]^2 + 4k \frac{P_0}{Q_0} \left[\frac{1}{n_a} + \frac{1}{e} \right] \right]^{0.5}}{\frac{2P_0}{Q_0} \left[\frac{1}{n_a} + \frac{1}{e} \right]} \\
 &= \frac{Q_0}{2} + \left[\frac{\left[P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] \right]^2 + 4k \frac{P_0}{Q_0} \left[\frac{1}{n_a} + \frac{1}{e} \right]}{\frac{4P_0^2}{Q_0^2} \left[\frac{1}{n_a} + \frac{1}{e} \right]^2} \right]^{0.5} \\
 &= \frac{Q_0}{2} + \left[\frac{P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] \left[P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] + \frac{4k}{Q_0} \right]}{P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] \left[\frac{4P_0}{Q_0^2} \left[\frac{1}{n_a} + \frac{1}{e} \right] \right]} \right]^{0.5} \\
 &= \frac{Q_0}{2} + \left[\frac{Q_0^2 \left[P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] + \frac{4k}{Q_0} \right]}{4P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right]} \right]^{0.5} \\
 &= \frac{Q_0}{2} + \left[\frac{Q_0^2 P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right]}{4P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right]} + \frac{\frac{4k}{Q_0} \cdot Q_0^2}{4P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right]} \right]^{0.5}
 \end{aligned}$$

$$= \frac{Q_0}{2} + \left[\frac{Q_0^2}{4} + \frac{kQ_0}{P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right]} \right]^{0.5}$$

Therefore,

$$(4.24b) \quad Q_1 = \frac{Q_0}{2} + Z^{0.5}$$

$$\text{where } Z = \frac{Q_0^2}{4} + \frac{kQ_0}{P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right]}$$

To find equation (4.23b), take equation (4.21)

$$(A13.1) \quad P_{d1} = a_a - b_a Q_1 + \frac{k}{Q_1}$$

Now substitute equation (4.24b) into (A13.1)

Therefore,

$$P_{d1} = P_0 \left[1 + \frac{1}{n_a} \right] - \frac{P_0}{Q_0} \cdot \frac{1}{n_a} \cdot \frac{Q_0}{2} - \frac{P_0}{Q_0} \cdot \frac{1}{n_a} \cdot Z^{0.5} + \frac{k}{\frac{Q_0}{2} + Z^{0.5}}$$

$$\text{where } Z = \frac{Q_0^2}{4} + \frac{kQ_0}{P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right]}$$

Therefore,

$$\begin{aligned} P_{d1} &= P_0 \left[1 + \frac{1}{n_a} \right] - P_0 \cdot \frac{1}{2n_a} - \frac{P_0}{Q_0} \cdot \frac{1}{n_a} \cdot Z^{0.5} + \frac{k}{\frac{Q_0}{2} + Z^{0.5}} \\ &= P_0 \left[1 + \frac{1}{2n_a} \right] - \frac{P_0}{Q_0} \cdot \frac{1}{n_a} \cdot Z^{0.5} + \frac{k}{\frac{1}{2}Q_0 + Z^{0.5}} \end{aligned}$$

That is,

$$(4.23b) \quad P_{d1} = P_0 \left[1 + \frac{1}{2n_a} \right] - \frac{P_0 Z^{0.5}}{Q_0 n_a} + \frac{k}{\frac{1}{2}Q_0 + Z^{0.5}}$$

$$\text{where } Z = \frac{Q_0^2}{4} + \frac{kQ_0}{P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right]}$$

A14. PROOF TO EQUATION (4.25a)

$$(4.25a) \quad \Delta Q = \frac{-(a_a - c) + X^{0.5}}{2(b_a + d)}$$

$$\text{where } X = (a_a - c)^2 + 4k(b_a + d)$$

From equations (4.24a) and (4.5b)

$$(4.24a) \quad Q_1 = \frac{a_a - c + X^{0.5}}{2(b_a + d)}$$

and

$$(4.5b) \quad Q_0 = \frac{a_a - c}{b_a + d}$$

Now,

$$(A14.1) \quad \Delta Q = Q_1 - Q_0$$

Therefore,

$$(4.25a) \quad \Delta Q = \frac{-(a_a - c) + X^{0.5}}{2(b_a + d)}$$

A15. PROOF TO EQUATION (4.25b)

$$(4.25b) \quad \Delta Q = -\frac{Q_0}{2} + Z^{0.5}$$

$$\text{where } Z = \frac{Q_0^2}{4} + \frac{kQ_0}{P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right]}$$

Take equation (4.25a)

$$(4.25a) \quad \Delta Q = \frac{-(a_a - c) + X^{0.5}}{2(b_a + d)}$$

From the proof to equation (4.24b), the proof to equation (4.25b) is directly analogous. That is,

$$(4.25b) \quad \Delta Q = -\frac{Q_0}{2} + Z^{0.5}$$

$$\text{where } Z = \frac{Q_0^2}{4} + \frac{kQ_0}{P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right]}$$

A16. PROOF TO EQUATION (4.30)

$$(4.30) \quad s_{11d} = \frac{n_i}{n_a} \cdot s_i + \frac{1}{2} \cdot \frac{Q_i}{Q_1} \left[1 - \frac{n_i}{n_a} \right]$$

Recall equation (4.29)

$$(4.29) \quad Q_{11d} = \frac{2b_a Q_i + a_i - a_a}{2b_i}$$

Substitute equations (4.17a) to (4.17d) into equation (4.29). This gives,

$$\begin{aligned} Q_{11d} &= \frac{2 \cdot \frac{1}{n_a} \cdot \frac{P_0}{Q_0} \cdot Q_i + P_0 + \frac{P_0}{n_i} - P_0 - \frac{P_0}{n_a}}{2 \cdot \frac{1}{n_i} \cdot \frac{P_0}{Q_i}} \\ &= \frac{\frac{2}{n_a} \cdot \frac{P_0}{Q_0} \cdot Q_i}{2 \cdot \frac{1}{n_i} \cdot \frac{P_0}{Q_i}} + \frac{P_0 \left[\frac{1}{n_i} - \frac{1}{n_a} \right]}{2 \left[\frac{1}{n_i} \cdot \frac{P_0}{Q_i} \right]} \end{aligned}$$

Therefore,

$$(A16.1) \quad Q_{11d} = \frac{n_i}{n_a} \cdot \frac{Q_i}{Q_0} \cdot Q_i + \frac{n_i Q_i}{2} \left[\frac{1}{n_i} - \frac{1}{n_a} \right]$$

$$(A16.2) \quad s_{11d} = \frac{Q_{11d}}{Q_i}$$

Substitute equation (A16.1) into equation (A16.2)

$$\begin{aligned} s_{11d} &= \frac{n_i}{n_a} \cdot \frac{Q_i}{Q_a} \cdot \frac{Q_i}{Q_i} + \frac{n_i Q_i}{2} \cdot \frac{1}{Q_i} \left[\frac{1}{n_i} - \frac{1}{n_a} \right] \\ &= \frac{n_i}{n_a} \cdot s_i + \frac{n_i Q_i}{2 Q_i n_i} - \frac{n_i Q_i}{2 Q_i n_a} \\ &= \frac{n_i}{n_a} \cdot s_i + \frac{Q_i}{2 Q_i} - \frac{Q_i}{2 Q_i} \cdot \frac{n_i}{n_a} \end{aligned}$$

Therefore,

$$(4.30) \quad s_{11d} = \frac{n_i}{n_a} \cdot s_i + \frac{1}{2} \cdot \frac{Q_i}{Q_i} \left[1 - \frac{n_i}{n_a} \right]$$

A17. PROOF TO EQUATION (4.31b)

$$(4.31b) \quad P_{11d} = P_0 \left[1 + \frac{1}{2n_i} + \frac{1}{n_a} \left[\frac{1}{2} - \frac{Q_i}{Q_0} \right] \right]$$

Recall equation (4.31a)

$$(4.31a) \quad P_{11d} = \frac{1}{2}(a_i + a_a) - b_a Q_i$$

Substitute equations (4.17a), (4.17c) and (4.17d) into equation (4.31a)

Therefore,

$$P_{11d} = \frac{1}{2} \left[P_0 + \frac{P_0}{n_i} + P_0 + \frac{P_0}{n_a} \right] - \frac{1}{n_a} \cdot \frac{P_0}{Q_0} \cdot Q_i$$

$$\begin{aligned}
 &= \frac{1}{2} \left[2P_0 + P_0 \left[\frac{1}{n_i} + \frac{1}{n_a} \right] \right] - \frac{1}{n_a} \cdot \frac{P_0}{Q_0} \cdot Q_1 \\
 &= P_0 + \frac{P_0}{2} \cdot \frac{1}{n_i} + \frac{P_0}{2} \cdot \frac{1}{n_a} - P_0 \cdot \frac{1}{n_a} \cdot \frac{Q_1}{Q_0} \\
 &= P_0 + \frac{P_0}{2} \cdot \frac{1}{n_i} + \frac{P_0}{n_a} \left[\frac{1}{2} - \frac{Q_1}{Q_0} \right]
 \end{aligned}$$

Therefore,

$$(4.31b) \quad P_{11d} = P_0 \left[1 + \frac{1}{2n_i} + \frac{1}{n_a} \left[\frac{1}{2} - \frac{Q_1}{Q_0} \right] \right]$$

A18. PROOF TO EQUATIONS (4.32c) and (4.32d)

Begin with

$$(4.32c) \quad \Delta TR = k + \frac{(b_a + d)[a_a X^{0.5} - 2b_a k - a_a(a_a - c)] + b_a(a_a - c)ctd}{2(b_a + d)^2}$$

$$\underline{(a_a - c - X^{0.5})}$$

$$\text{where } X = (a_a - c)^2 + 4k(b_a + d)$$

Begin with equation (4.32b)

$$(4.32b) \quad \Delta TR = k + (P_1 Q_1 - P_0 Q_0)$$

Recall equations (4.5a), (4.5b) (4.24) and (4.33)

$$(4.5a) \quad Q_0 = \frac{a_a - c}{b_a + d}$$

$$(4.5b) \quad P_0 = a_a - b_a \left[\frac{a_a - c}{b_a + d} \right]$$

$$(4.24) \quad Q_1 = \frac{a_a - c + X^{0.5}}{2(b_a + d)}$$

$$(4.33) \quad P_1 = a_a - b_a \left[\frac{a_a - c + X^{0.5}}{2(b_a + d)} \right]$$

$$\text{where } X = (a_a - c)^2 + 4k(b_a + d)$$

ΔTR can be found by substituting these equations into (4.32b)

Find $P_1 Q_1$ by multiplying (4.24) by (4.33)

Therefore,

$$\begin{aligned} P_1 Q_1 &= \left[\frac{a_a - c + X^{0.5}}{2(b_a + d)} \right] \left[a_a - b_a \left[\frac{a_a - c + X^{0.5}}{2(b_a + d)} \right] \right] \\ &= \frac{a_a(a_a - c + X^{0.5})}{2(b_a + d)} - \frac{b_a(a_a - c + X^{0.5})^2}{4(b_a + d)^2} \\ &= \frac{a_a(a_a - c)}{2(b_a + d)} + \frac{a_a X^{0.5}}{2(b_a + d)} - \frac{b_a(a_a - c + X^{0.5})(a_a - c + X^{0.5})}{4(b_a + d)^2} \\ &= \frac{a_a(a_a - c)}{2(b_a + d)} + \frac{a_a X^{0.5}}{2(b_a + d)} \\ &\quad - \frac{b_a(a_a^2 - a_a c + a_a X^{0.5} - a_a c + c^2 - c X^{0.5} + a_a X^{0.5} - c X^{0.5} + X)}{4(b_a + d)^2} \\ &= \frac{a_a(a_a - c)}{2(b_a + d)} + \frac{a_a X^{0.5}}{2(b_a + d)} \\ &\quad - \frac{b_a[(a_a - c)^2 + 2a_a X^{0.5} - 2c X^{0.5} + X]}{4(b_a + d)^2} \\ &= \frac{a_a(a_a - c)}{2(b_a + d)} + \frac{a_a X^{0.5}}{2(b_a + d)} - \frac{b_a(a_a - c)^2}{4(b_a + d)^2} \end{aligned}$$

$$- \frac{2b_a X^{0.5}(a_a - c)}{4(b_a + d)^2} - \frac{b_a X}{4(b_a + d)^2}$$

Take term 5

$$\begin{aligned} - \frac{b_a X}{4(b_a + d)^2} &= \frac{-b_a[(a_a - c)^2 + 4k(b_a + d)]}{4(b_a + d)^2} \\ &= \frac{-b_a(a_a - c)^2}{4(b_a + d)^2} - \frac{4b_a k(b_a + d)}{4(b_a + d)^2} \end{aligned}$$

Therefore,

$$\begin{aligned} \text{(A18.1)} \quad P_1 Q_1 &= \frac{a_a(a_a - c)}{2(b_a + d)} + \frac{a_a X^{0.5}}{2(b_a + d)} - \frac{2b_a(a_a - c)}{4(b_a + d)} \\ &\quad - \frac{2b_a X^{0.5}(a_a - c)}{4(b_a + d)^2} - \frac{b_a k}{(b_a + d)} \end{aligned}$$

Find $P_0 Q_0$ by multiplying (4.5a) by (4.5b)

Therefore,

$$P_0 Q_0 = \left[\frac{a_a - c}{b_a + d} \right] \left[a_a - b_a \left[\frac{a_a - c}{b_a + d} \right] \right]$$

That is,

$$\text{(A18.2)} \quad P_0 Q_0 = \frac{a_a(a_a - c)}{(b_a + d)} - \frac{b_a(a_a - c)^2}{(b_a + d)^2}$$

Substituting (A18.1) and (A18.2) into (4.32b) gives

$$\begin{aligned} \Delta TR &= k + \frac{a_a(a_a - c)}{2(b_a + d)} + \frac{a_a X^{0.5}}{2(b_a + d)} - \frac{2b_a(a_a - c)^2}{4(b_a + d)^2} - \frac{2b_a X^{0.5}(a_a - c)}{4(b_a + d)^2} \\ &\quad - \frac{b_a k}{(b_a + d)} - \frac{a_a(a_a - c)}{(b_a + d)} + \frac{b_a(a_a - c)^2}{(b_a + d)^2} \end{aligned}$$

Therefore,

$$\text{(A18.3)} \quad \Delta TR = k - \frac{a_a(a_a - c)}{2(b_a + d)} + \frac{b_a(a_a - c)^2}{2(b_a + d)^2} - \frac{b_a k}{(b_a + d)} + \frac{a_a X^{0.5}}{2(b_a + d)}$$

$$- \frac{b_a X^{0.5} (a_a - c)}{2(b_a + d)^2}$$

That is,

(4.32c)

$$\Delta TR = k + \frac{(b_a + d)[a_a X^{0.5} - 2b_a k - a_a(a_a - c)] + b_a(a_a - c)(a_a - c - X^{0.5})}{2(b_a + d)^2}$$

$$\text{where } X = (a_a - c)^2 + 4k(b_a + d)$$

Recall equation (4.32d)

$$(4.32d) \quad \Delta TR = k + \left[\frac{P_0^2 Q_0^2}{4} + P_0 Q_0 k \frac{n_a e}{n_a + e} \right]^{0.5}$$

$$- \frac{P_0 Q_0}{2} - k \frac{e}{n_a + e}$$

Substitute equations (4.17a) to (4.17f) into equation (A18.3)

Take term 2

$$- \frac{a_a(a_a - c)}{2(b_a + d)}$$

$$= \frac{- \left[P_0 + \frac{P_0}{n_a} \right] \left[P_0 + \frac{P_0}{n_a} - P_0 + \frac{P_0}{e} \right]}{2 \left[\frac{1}{n_a} \cdot \frac{P_0}{Q_0} + \frac{1}{e} \cdot \frac{P_0}{Q_0} \right]}$$

$$= \frac{-P_0 \left[1 + \frac{1}{n_a} \right] P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right]}{\frac{2P_0}{Q_0} \left[\frac{1}{n_a} + \frac{1}{e} \right]}$$

$$= - \frac{P_0 Q_0}{2} \left[1 + \frac{1}{n_a} \right]$$

Take term 3

$$\begin{aligned}
 & \frac{b_a(a_a - c)^2}{2(b_a + d)^2} \\
 & \frac{\frac{1}{n_a} \cdot \frac{P_o}{Q_o} \left[\frac{P_o}{n_a} + \frac{P_o}{e} \right]^2}{2 \left[\frac{1}{n_a} \cdot \frac{P_o}{Q_o} + \frac{1}{e} \cdot \frac{P_o}{Q_o} \right]^2} \\
 & \frac{\frac{1}{n_a} \cdot \frac{P_o}{Q_o} \cdot P_o^2 \left[\frac{1}{n_a} + \frac{1}{e} \right]^2}{2 \frac{P_o^2}{Q_o^2} \left[\frac{1}{n_a} + \frac{1}{e} \right]^2} \\
 & = \frac{P_o Q_o}{2} \cdot \frac{1}{n_a}
 \end{aligned}$$

Take term 4

$$\begin{aligned}
 & - \frac{b_a k}{(b_a + d)} \\
 & - \frac{\frac{1}{n_a} \cdot \frac{P_o}{Q_o} \cdot k}{\left[\frac{1}{n_a} \cdot \frac{P_o}{Q_o} + \frac{1}{e} \cdot \frac{P_o}{Q_o} \right]} \\
 & - \frac{\frac{P_o}{Q_o} \cdot \frac{1}{n_a} \cdot k}{\frac{P_o}{Q_o} \left[\frac{1}{n_a} + \frac{1}{e} \right]}
 \end{aligned}$$

$$= - \frac{\frac{1}{n_a} k}{\left[\frac{1}{n_a} + \frac{1}{e} \right]}$$

Take term 5

$$\begin{aligned} & \frac{a_a X^{0.5}}{2(b_a + d)} \\ &= \frac{a_a \{ (a_a - c)^2 + 4k(b_a + d) \}^{0.5}}{2(b_a + d)} \\ &= \frac{\left[P_0 + \frac{P_0}{n_a} \right] \left[\left[\frac{P_0}{n_a} + \frac{P_0}{e} \right]^2 + 4k \left[\frac{1}{n_a} \cdot \frac{P_0}{Q_0} + \frac{1}{e} \cdot \frac{P_0}{Q_0} \right] \right]^{0.5}}{2 \left[\frac{1}{n_a} \cdot \frac{P_0}{Q_0} + \frac{1}{e} \cdot \frac{P_0}{Q_0} \right]} \\ &= \frac{\left[P_0 + \frac{P_0}{n_a} \right] \left[\left[P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] \right]^2 + 4k \frac{P_0}{Q_0} \left[\frac{1}{n_a} + \frac{1}{e} \right] \right]^{0.5}}{\frac{2P_0}{Q_0} \left[\frac{1}{n_a} + \frac{1}{e} \right]} \\ &= \frac{P_0 \left[1 + \frac{1}{n_a} \right] \left[P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] \left[P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] + \frac{4k}{Q_0} \right] \right]^{0.5}}{\frac{2P_0}{Q_0} \left[\frac{1}{n_a} + \frac{1}{e} \right]} \\ &= \frac{Q_0 \left[1 + \frac{1}{n_a} \right] \left[P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] \right]^{0.5} \left[P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] + \frac{4k}{Q_0} \right]^{0.5}}{2 \left[\frac{1}{n_a} + \frac{1}{e} \right]} \end{aligned}$$

Take term 6

$$\begin{aligned}
 & - \frac{b_a X^{0.5}(a_a - c)}{2(b_a + d)^2} \\
 & - \frac{b_a \{(a_a - c)^2 + 4k(b_a + d)\}^{0.5}(a_a - c)}{2(b_a + d)^2} \\
 & = \frac{-\frac{1}{n_a} \cdot \frac{P_0}{Q_0} \left[\frac{P_0}{n_a} + \frac{P_0}{e} \right]^2 + 4k \left[\frac{1}{n_a} \cdot \frac{P_0}{Q_0} + \frac{1}{e} \cdot \frac{P_0}{Q_0} \right]^{0.5} \left[\frac{P_0}{n_a} + \frac{P_0}{e} \right]}{2 \left[\frac{1}{n_a} \cdot \frac{P_0}{Q_0} + \frac{1}{e} \cdot \frac{P_0}{Q_0} \right]^2} \\
 & = \frac{-\frac{1}{n_a} \cdot \frac{P_0}{Q_0} \left[P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] \right]^{0.5} \cdot \left[P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] + \frac{4k}{Q_0} \right]^{0.5} \cdot P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right]}{2 \frac{P_0^2}{Q_0^2} \left[\frac{1}{n_a} + \frac{1}{e} \right]^2} \\
 & = \frac{-\frac{1}{n_a} Q_0 \left[P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] \right]^{0.5} \cdot \left[P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] + \frac{4k}{Q_0} \right]^{0.5}}{2 \left[\frac{1}{n_a} + \frac{1}{e} \right]}
 \end{aligned}$$

Put all terms back together again.

This gives

$$\begin{aligned}
 \Delta TR = k - & \frac{P_0 Q_0}{2} \left[1 + \frac{1}{n_a} \right] + \frac{P_0 Q_0}{2} \cdot \frac{1}{n_a} - \frac{\frac{1}{n_a} \cdot k}{\left[\frac{1}{n_a} + \frac{1}{e} \right]} \\
 & + \frac{Q_0 \left[1 + \frac{1}{n_a} \right] \left[P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] \right]^{0.5} \left[P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] + \frac{4k}{Q_0} \right]^{0.5}}{2 \left[\frac{1}{n_a} + \frac{1}{e} \right]}
 \end{aligned}$$

$$\begin{aligned}
& - \frac{\frac{1}{n_a} Q_0 \left[P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] \right]^{0.5} \left[P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] + \frac{4k}{Q_0} \right]^{0.5}}{2 \left[\frac{1}{n_a} + \frac{1}{e} \right]} \\
& = k + \frac{P_0 Q_0}{2} \left[\frac{1}{n_a} - 1 - \frac{1}{n_a} \right] - \frac{\frac{1}{n_a} \cdot k}{\left[\frac{1}{n_a} + \frac{1}{e} \right]} \\
& + \frac{\left[P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] \right]^{0.5} \left[P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] + \frac{4k}{Q_0} \right]^{0.5} \left[Q_0 \left[1 + \frac{1}{n_a} \right] - \frac{Q_0}{n_a} \right]}{2 \left[\frac{1}{n_a} + \frac{1}{e} \right]} \\
& = k - \frac{P_0 Q_0}{2} - \frac{\frac{1}{n_a} k}{\left[\frac{1}{n_a} + \frac{1}{e} \right]} + \frac{P_0^{0.5} \cdot \left[\frac{1}{n_a} + \frac{1}{e} \right]^{0.5} \left[P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] + \frac{4k}{Q_0} \right]^{0.5} \cdot Q_0}{2 \left[\frac{1}{n_a} + \frac{1}{e} \right]^{0.5} \left[\frac{1}{n_a} + \frac{1}{e} \right]^{0.5}} \\
& = k - \frac{P_0 Q_0}{2} - \frac{k}{n_a \left[\frac{1}{n_a} + \frac{1}{e} \right]} + \frac{\left[P_0 \left[P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] + \frac{4k}{Q_0} \right] \right]^{0.5} \cdot Q_0}{2 \left[\frac{1}{n_a} + \frac{1}{e} \right]^{0.5}} \\
& = k - \frac{P_0 Q_0}{2} - \frac{k}{\frac{n_a (n_a + e)}{n_a e}} + \frac{Q_0}{2} \left[\frac{P_0^2 \left[\frac{1}{n_a} + \frac{1}{e} \right]}{\left[\frac{1}{n_a} + \frac{1}{e} \right]} + \frac{4k P_0}{Q_0 \left[\frac{1}{n_a} + \frac{1}{e} \right]} \right]^{0.5}
\end{aligned}$$

$$\begin{aligned}
 &= k - \frac{P_0 Q_0}{2} - \frac{ek}{n_a + e} + \frac{Q_0}{2} \left[P_0^2 + \frac{4kP_0}{Q_0 \left[\frac{1}{n_a} + \frac{1}{e} \right]} \right]^{0.5} \\
 &= k + \left[\frac{P_0^2 Q_0^2}{4} + \frac{4kP_0 Q_0^2}{4Q_0} \cdot \frac{n_a e}{(n_a + e)} \right]^{0.5} - \frac{P_0 Q_0}{2} - \frac{ke}{(n_a + e)} \\
 &= k + \left[\frac{P_0^2 Q_0^2}{4} + P_0 Q_0 k \frac{n_a e}{(n_a + e)} \right]^{0.5} - \frac{P_0 Q_0}{2} - \frac{ke}{(n_a + e)}
 \end{aligned}$$

Therefore,

$$\begin{aligned}
 (4.32d) \quad \Delta TR &= k + \left[\frac{P_0^2 Q_0^2}{4} + P_0 Q_0 k \frac{n_a e}{(n_a + e)} \right]^{0.5} \\
 &\quad - \frac{P_0 Q_0}{2} - k \frac{e}{(n_a + e)}
 \end{aligned}$$

A19. PROOF TO EQUATION (4.35a)

$$(4.35a) \quad \Delta C = c(Q_1 - Q_0) + \frac{d}{2}(Q_1^2 - Q_0^2)$$

Now

$$\begin{aligned}
 (A19.1) \quad \Delta C &= \int_{Q_0}^{Q_1} (c + dQ) dQ \\
 &= \left[cQ + \frac{d}{2} Q^2 \right]_{Q_0}^{Q_1} \\
 &= cQ_1 + \frac{d}{2} Q_1^2 - cQ_0 - \frac{d}{2} Q_0^2
 \end{aligned}$$

That is,

$$\Delta C = c(Q_1 - Q_0) + \frac{d}{2}(Q_1^2 - Q_0^2)$$

A20. PROOF TO EQUATION (4.35b)

$$(4.35b) \quad \Delta C = \frac{2(b_a + d) \left[-c(a_a - c) + X^{0.5} \right] ctd}{4(b_a + d)^2}$$

$$= \frac{\frac{3d}{2}(a_a - c)^2 + d(a_a - c)X^{0.5} + \frac{d}{2}X}{4(b_a + d)^2}$$

$$\text{where } X = (a_a - c)^2 + 4k(b_a + d)$$

From (A19.1)

$$(A19.1) \quad \int_{Q_0}^{Q_1} (c + dQ) dQ = c(Q_1 - Q_0) + \frac{d}{2} (Q_1^2 - Q_0^2)$$

Substitute equations (4.5b) and (4.24) into (A19.1)

$$\begin{aligned} \Delta C &= c \left[\frac{a_a - c + X^{0.5}}{2(b_a + d)} - \frac{a_a - c}{b_a + d} \right] + \frac{d}{2} \left[\left(\frac{a_a - c + X^{0.5}}{2(b_a + d)} \right)^2 - \frac{(a_a - c)^2}{(b_a + d)^2} \right] \\ &= c \left[\frac{(a_a - c) + X^{0.5} - 2(a_a - c)}{2(b_a + d)} \right] + \frac{d}{2} \left[\frac{(a_a - c + X^{0.5})^2}{4(b_a + d)^2} - \frac{4(a_a - c)^2}{4(b_a + d)^2} \right] \\ &= c \left[\frac{-(a_a - c) + X^{0.5}}{2(b_a + d)} \right] + \frac{d}{2} \left[\frac{(a_a - c)^2 + 2(a_a - c)X^{0.5} + X - 4(a_a - c)^2}{4(b_a + d)^2} \right] \end{aligned}$$

That is,

$$(A20.1) \quad \Delta C = \frac{-c[(a_a - c) + X^{0.5}]}{2(b_a + d)} + \frac{-\frac{3d}{2}(a_a - c)^2 + d(a_a - c)X^{0.5} + \frac{d}{2}X}{4(b_a + d)^2}$$

Therefore,

$$(4.35b) \quad \Delta C = \frac{2(b_a + d) \left[-c(a_a - c) + X^{0.5} \right] ctd}{4(b + d)^2} - \frac{\frac{3d}{2}(a_a - c)^2 + d(a_a - c)X^{0.5} + \frac{d}{2}X}{4(b + d)^2}$$

A21. PROOF TO EQUATION (4.34b)

$$(4.34b) \quad \Delta PS = \frac{-d(a_a - c)^2 + d(a_a - c)X^{0.5} + 2dk(b_a + d)}{4(b_a + d)^2}$$

$$\text{where } X = (a_a - c)^2 + 4k(b_a + d)$$

Combine equations (A18.3) and (A19.1)

This gives

$$\begin{aligned} \Delta PS &= k - \frac{a_a(a_a - c)}{2(b_a + d)} + \frac{a_a X^{0.5}}{2(b_a + d)} - \frac{2b_a k}{2(b_a + d)} + \frac{b_a(a_a - c)^2}{2(b_a + d)^2} \\ &\quad - \frac{b_a X^{0.5}(a_a - c)}{2(b_a + d)^2} - \frac{-c(a_a - c) + cX^{0.5}}{2(b_a + d)} \\ &\quad - \frac{\frac{3d}{2}(a_a - c)^2 + d(a_a - c)X^{0.5} + \frac{d}{2}X}{4(b_a + d)^2} \\ &= \frac{2k(b_a + d) - a_a(a_a - c) + a_a X^{0.5} - 2b_a k + c(a_a - c) - cX^{0.5}}{2(b_a + d)} \\ &\quad + \frac{2b_a(a_a - c)^2 - 2b_a X^{0.5}(a_a - c) + \frac{3d}{2}(a_a - c)^2 - d(a_a - c)X^{0.5} - \frac{d}{2}X}{4(b_a + d)^2} \end{aligned}$$

$$\begin{aligned}
&= \frac{2k(b_a+d) - 2b_a k - a_a(a_a-c) + c(a_a-c) + a_a X^{0.5} - c X^{0.5}}{2(b_a+d)} \\
&+ \frac{2b_a(a_a-c)^2 + \frac{3}{2}d(a_a-c)^2 - 2b X^{0.5}(a_a-c) - d X^{0.5}(a_a-c) - \frac{d}{2}X}{4(b_a+d)^2} \\
&= \frac{2k(b_a+d-b_a) - (a_a-c)(a_a-c) + X^{0.5}(a_a-c)}{2(b_a+d)} \\
&+ \frac{(a_a-c)^2(2b_a + \frac{3}{2}d) - (a_a-c)X^{0.5}(2b_a+d) - ctd}{4(b_a+d)^2} \\
&\frac{d}{2} \left[(a_a-c)^2 + 4k(b_a+d) \right] \\
&= \frac{2dk - (a_a-c)^2 + X^{0.5}(a_a-c)}{2(b_a+d)} \\
&+ \frac{2b_a(a_a-c)^2 + \frac{3}{2}d(a_a-c)^2 - \frac{d}{2}(a_a-c)^2 ctd}{4(b_a+d)^2} \\
&- \frac{(a_a-c)X^{0.5}(2b_a+d) - 2dk(b_a+d)}{4(b_a+d)^2} \\
&= \frac{4dk(b_a+d) - 2(a_a-c)^2(b_a+d) + 2X^{0.5}(a_a-c)(b_a+d) ctd}{4(b_a+d)^2} \\
&+ \frac{(2b_a+d)(a_a-c)^2 - (2b_a+d)(a_a-c)X^{0.5} - 2dk(b_a+d)}{4(b_a+d)^2} \\
&= \frac{(a_a-c)^2(2b_a+d-2b_a-2d) + X^{0.5}(a_a-c)(2b_a+2d-2b_a-d) ctd}{4(b_a+d)^2} \\
&+ \frac{2dk(b_a+d)}{4(b_a+d)^2}
\end{aligned}$$

Therefore,

$$(4.34b) \quad \Delta PS = \frac{-d(a_a - c)^2 + d(a_a - c)X^{0.5} + 2dk(b_a + d)}{(4b_a + d)^2}$$

A22. PROOF TO EQUATION (4.34c)

$$(4.34c) \quad \Delta PS = \frac{1}{4e} \left[\frac{2k}{\left[\frac{1}{n_a} + \frac{1}{e} \right]} - P_0 Q_0 + \left[P_0^2 Q_0^2 + 4k P_0 Q_0 \frac{1}{\left[\frac{1}{n_a} + \frac{1}{e} \right]} \right]^{0.5} \right]$$

Substitute equations (4.17a) to (4.17f) into (4.34b)

Take term 1 of the numerator in (4.34b)

$$\begin{aligned} -d(a_a - c)^2 &= -\frac{1}{e} \cdot \frac{P_0}{Q_0} \left[\frac{P_0}{n_a} + \frac{P_0}{e} \right]^2 \\ &= -\frac{1}{e} \cdot \frac{P_0}{Q_0} \cdot P_0^2 \left[\frac{1}{n_a} + \frac{1}{e} \right]^2 \end{aligned}$$

Take term 2 of the numerator in (4.34b)

$$d(a_a - c)X^{0.5} = \frac{1}{e} \cdot \frac{P_0}{Q_0} \cdot P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] \left[P_0^2 \left[\frac{1}{n_a} + \frac{1}{e} \right]^2 + 4k \frac{P_0}{Q_0} \left[\frac{1}{n_a} + \frac{1}{e} \right] \right]^{0.5}$$

Take term 3 of the numerator in (4.34b)

$$2dk(b_a + d) = 2k \frac{1}{e} \cdot \frac{P_0^2}{Q_0^2} \left[\frac{1}{n_a} + \frac{1}{e} \right]$$

Take the denominator in (4.34b)

$$4(b_a + d)^2 = 4 \frac{P_0^2}{Q_0^2} \left[\frac{1}{n_a} + \frac{1}{e} \right]^2$$

Put all terms back together

$$\Delta PS = \frac{-\frac{1}{e} \cdot \frac{P_0}{Q_0} \cdot P_0^2 \left[\frac{1}{n_a} + \frac{1}{e} \right]^2 + 2k \cdot \frac{1}{e} \cdot \frac{P_0^2}{Q_0^2} \left[\frac{1}{n_a} + \frac{1}{e} \right] \text{ ctd}}{\frac{4P_0^2}{Q_0^2} \left[\frac{1}{n_a} + \frac{1}{e} \right]^2} + \frac{1}{e} \cdot \frac{P_0}{Q_0} \cdot P_0 \left[\frac{1}{n_a} + \frac{1}{e} \right] \left[P_0^2 \left[\frac{1}{n_a} + \frac{1}{e} \right]^2 + 4k \frac{P_0}{Q_0} \left[\frac{1}{n_a} + \frac{1}{e} \right] \right]^{0.5}}$$

$$= -\frac{1}{4e} P_0 Q_0 + \frac{2}{4e} \cdot \frac{k}{\left[\frac{1}{n_a} + \frac{1}{e} \right]} \text{ ctd} + \frac{1}{4e} \cdot \frac{P_0^2}{Q_0} \left[\frac{1}{n_a} + \frac{1}{e} \right] \left[\frac{P_0^2 \left[\frac{1}{n_a} + \frac{1}{e} \right]^2 + 4k \frac{P_0}{Q_0} \left[\frac{1}{n_a} + \frac{1}{e} \right]}{\frac{P_0^4}{Q_0^4} \left[\frac{1}{n_a} + \frac{1}{e} \right]^4} \right]^{0.5}$$

Simplify the last term of this expression

$$\frac{1}{4e} \cdot \frac{P_0^2}{Q_0^2} \left[\frac{1}{n_a} + \frac{1}{e} \right] \left[\frac{P_0^2 Q_0^4}{P_0^4} \cdot \frac{1}{\left[\frac{1}{n_a} + \frac{1}{e} \right]^2} + \frac{P_0 Q_0^4}{P_0^4 Q_0} \cdot \frac{4k}{\left[\frac{1}{n_a} + \frac{1}{e} \right]^3} \right]^{0.5}$$

$$= \frac{1}{4e} \cdot \frac{P_0^2}{Q_0} \left[\frac{1}{n_a} + \frac{1}{e} \right] \cdot \frac{Q_0}{P_0^2} \cdot \frac{1}{\left[\frac{1}{n_a} + \frac{1}{e} \right]} \left[P_0^2 Q_0^2 + \frac{P_0 Q_0^2}{Q_0} \cdot \frac{4k}{\left[\frac{1}{n_a} + \frac{1}{e} \right]} \right]^{0.5}$$

$$= \frac{1}{4e} \left[P_0^2 Q_0^2 + 4k P_0 Q_0 \frac{1}{\left[\frac{1}{n_a} + \frac{1}{e} \right]} \right]^{0.5}$$

Therefore,

$$(4.34c) \Delta PS = \frac{1}{4e} \left[\frac{2k}{\left[\frac{1}{n_a} + \frac{1}{e} \right]} - P_0 Q_0 + \left[P_0^2 Q_0^2 + 4kP_0 Q_0 \frac{1}{\left[\frac{1}{n_a} + \frac{1}{e} \right]} \right]^{0.5} \right]$$

APPENDIX 2PROOFS TO SELECTED EQUATIONS IN CHAPTER 5A22. DERIVATION OF EQUATION (5.10a) AND (5.10b) FROM EQUATION (5.9)

Take equation (5.9)

$$(5.9) \quad \frac{\Theta_2 - \beta_2}{\Theta_1 - \beta_1} = \frac{P_1 \beta_2}{P_2 \beta_1}$$

Assume

$$(A22.1) \quad P_1 > P_2$$

$$(A22.2) \quad \beta_1 = \beta_2 = \beta^*$$

and

$$(A22.3) \quad \beta^* < \Theta_1, \Theta_2$$

Therefore,

$$(A22.4) \quad \frac{\beta^* P_1}{\beta^* P_2} = \frac{\Theta_2 - \beta^*}{\Theta_1 - \beta^*}$$

From equations (A23.1) and (A23.3), it follows that

$$(A22.5) \quad \beta^* P_1 > \beta^* P_2$$

and

$$(A22.6) \quad (\Theta_2 - \beta^*) > (\Theta_1 - \beta^*)$$

Therefore,

$$(5.10a) \quad \Theta_1 < \Theta_2$$

If, however,

$$(A22.7) \quad \beta^* > \theta_1, \theta_2$$

then

$$(A22.8) \quad (\theta_2 - \beta^*) < (\theta_1 - \beta^*)$$

and

$$(5.10b) \quad \theta_1 > \theta_2$$

A23. COMMENT ON EQUATIONS (5.11a) AND (5.11b)

If

$$(A23.1) \quad \pi = p_1 q_1 + p_2 q_2 - C(q) - A_1 - A_2$$

It can be shown that

$$(A23.2) \quad \theta_1 = \frac{\beta_1}{n_1}$$

and

$$(A23.3) \quad \theta_2 = \frac{\beta_2}{n_2}$$

From (A22.1) and (A22.2)

$$(5.11a) \quad \theta_1 > \theta_2$$

If, however,

$$(A23.4) \quad \beta_1 < \beta_2$$

then it is not possible to determine from equations

(A23.2) and (A23.3) whether

$$(5.11b) \quad \theta_1 \begin{matrix} > \\ = \\ < \end{matrix} \theta_2$$

A24. PROOF TO EQUATION (5.22a)

$$(5.22a) \quad \frac{\delta R_1}{\delta A_1} = Q \frac{\delta P}{\delta A_1} - 1$$

Recall equation (5.20a)

$$(5.20a) \quad R_1 = PS - A_1$$

That is

$$(A24.1) \quad R_1 = PQ - \int_0^Q S^{-1}(Q) dQ - A_1$$

Therefore,

$$\begin{aligned} \frac{\delta R_1}{\delta A_1} &= \frac{\delta}{\delta A_1}(PQ) - \frac{\delta}{\delta A_1} \left[\int_0^Q S^{-1}(Q) dQ \right] - \frac{\delta A_1}{\delta A_1} \\ &= P \frac{\delta Q}{\delta A_1} + Q \frac{\delta P}{\delta A_1} - S^{-1}(Q) \frac{\delta Q}{\delta A_1} - 1 \end{aligned}$$

Now, $S^{-1}(Q) = P$. Therefore,

$$\frac{\delta R_1}{\delta A_1} = P \frac{\delta Q}{\delta A_1} + Q \frac{\delta P}{\delta A_1} - P \frac{\delta Q}{\delta A_1} - 1$$

That is,

$$(5.22a) \quad \frac{\delta R_1}{\delta A_1} = Q \frac{\delta P}{\delta A_1} - 1$$

A25. PROOF TO EQUATION (5.23a)

$$(5.23a) \quad \frac{\delta Q}{\delta A_1} - \frac{\delta D_1}{\delta P} \cdot \frac{\delta P}{\delta A_1} - \frac{\delta D_2}{\delta P} \cdot \frac{\delta P}{\delta A_1} = \frac{\delta D_1}{\delta A_1}$$

Recall equation (5.17a)

$$(5.17c) \quad Q_1 = D_1(P, A_1) + D_2(P, A_2)$$

Differentiating (5.17c) with respect to A_1 gives

$$\begin{aligned}
 \frac{\delta Q}{\delta A_1} &= \frac{\delta}{\delta A_1} [D_1(P, A_1) + D_2(P, A_2)] \\
 &= \frac{\delta}{\delta A_1} [D_1(P, A_1)] + \frac{\delta}{\delta A_1} [D_2(P, A_2)] \\
 &= \frac{\delta}{\delta A_1} [D_1(P)] + \frac{\delta}{\delta A_1} [D_1(A_1)] + \frac{\delta}{\delta A_1} [D_2(P)] + \frac{\delta}{\delta A_1} [D_2(A_2)] \\
 &= \frac{\delta D_1}{\delta P} \cdot \frac{\delta P}{\delta A_1} + \frac{\delta D_1}{\delta A_1} \cdot \frac{\delta A_1}{\delta A_1} + \frac{\delta D_2}{\delta P} \cdot \frac{\delta P}{\delta A_1} + \frac{\delta D_2}{\delta A_1} \cdot \frac{\delta A_2}{\delta A_1}
 \end{aligned}$$

Now, $\frac{\delta A_1}{\delta A_1} = 1$ and $\frac{\delta A_2}{\delta A_1} = 0$. Therefore,

$$\frac{\delta Q}{\delta A_1} = \frac{\delta D_1}{\delta P} \cdot \frac{\delta P}{\delta A_1} + \frac{\delta D_1}{\delta A_1} + \frac{\delta D_2}{\delta P} \cdot \frac{\delta P}{\delta A_1}$$

That is,

$$(5.23a) \quad \frac{\delta Q}{\delta A_1} - \frac{\delta D_1}{\delta P} \cdot \frac{\delta P}{\delta A_1} - \frac{\delta D_2}{\delta P} \cdot \frac{\delta P}{\delta A_1} = \frac{\delta D_1}{\delta A_1}$$

A26. PROOF TO EQUATION (5.23b)

$$(5.23b) \quad \frac{\delta Q}{\delta A_1} - \frac{\delta S}{\delta P} \cdot \frac{\delta P}{\delta A_1} = 0$$

Recall equation (5.18)

$$(5.18) \quad Q = S(P)$$

Differentiating (5.18) with respect to A_1 gives

$$\begin{aligned}
 \frac{\delta Q}{\delta A_1} &= \frac{\delta}{\delta A_1} [S(P)] \\
 &= \frac{\delta S}{\delta P} \cdot \frac{\delta P}{\delta A_1}
 \end{aligned}$$

Therefore,

$$(5.23b) \quad \frac{\delta Q}{\delta A_1} - \frac{\delta S}{\delta P} \cdot \frac{\delta P}{\delta A_1} = 0$$

A27. PROOF TO EQUATION (5.24a)

$$(5.24a) \quad \frac{\delta P}{\delta A_1} = \frac{\frac{\delta D_1}{\delta A_1}}{\frac{\delta S}{\delta P} - \left[\frac{\delta D_1}{\delta P} + \frac{\delta D_2}{\delta P} \right]}$$

Recall equations (5.23a) and (5.23b)

$$(5.23a) \quad \frac{\delta Q}{\delta A_1} - \frac{\delta D_1}{\delta P} \cdot \frac{\delta P}{\delta A_1} - \frac{\delta D_2}{\delta P} \cdot \frac{\delta P}{\delta A_1} = \frac{\delta D_1}{\delta A_1}$$

$$(5.23b) \quad \frac{\delta Q}{\delta A_1} - \frac{\delta S}{\delta P} \cdot \frac{\delta P}{\delta A_1} = 0$$

Subtract (5.23b) from (5.23a)

This gives

$$\frac{\delta Q}{\delta A_1} - \frac{\delta D_1}{\delta P} \cdot \frac{\delta P}{\delta A_1} - \frac{\delta D_2}{\delta P} \cdot \frac{\delta P}{\delta A_1} - \frac{\delta Q}{\delta A_1} + \frac{\delta S}{\delta P} \cdot \frac{\delta P}{\delta A_1} = \frac{\delta D_1}{\delta A_1}$$

That is,

$$- \frac{\delta D_1}{\delta P} \cdot \frac{\delta P}{\delta A_1} - \frac{\delta D_2}{\delta P} \cdot \frac{\delta P}{\delta A_1} + \frac{\delta S}{\delta P} \cdot \frac{\delta P}{\delta A_1} = \frac{\delta D_1}{\delta A_1}$$

and

$$\frac{\delta P}{\delta A_1} \left[\frac{\delta S}{\delta P} - \frac{\delta D_1}{\delta P} - \frac{\delta D_2}{\delta P} \right] = \frac{\delta D_1}{\delta A_1}$$

Therefore,

$$(5.24a) \quad \frac{\delta P}{\delta A_1} = \frac{\frac{\delta D_1}{\delta A_1}}{\frac{\delta S}{\delta P} - \left[\frac{\delta D_1}{\delta P} + \frac{\delta D_2}{\delta P} \right]}$$

A28. PROOF TO EQUATION (5.26a)

$$(5.26a) \quad \frac{\delta R_1}{\delta A_1} = \frac{\alpha_1}{e - (n_1 s_1 + n_2 s_2)} - 1$$

Recall equation (5.25a)

$$(5.25a) \quad \frac{\delta R_1}{\delta A_1} = Q \left[\frac{\frac{\delta D_1}{\delta A_1}}{\frac{\delta S}{\delta P} - \left[\frac{\delta D_1}{\delta P} + \frac{\delta D_2}{\delta P} \right]} \right] - 1$$

Multiply the right-hand side by $\left[\frac{P}{Q} \right] / \left[\frac{P}{Q} \right]$

Therefore,

$$\frac{\delta R_1}{\delta A_1} = \frac{\frac{P}{Q} \cdot Q \cdot \frac{\delta D_1}{\delta A_1}}{\frac{P}{Q} \cdot \frac{\delta S}{\delta P} - \left[\frac{P}{Q} \cdot \frac{Q}{Q_1} \cdot \frac{Q_1}{Q} \cdot \frac{\delta D_1}{\delta P} + \frac{P}{Q} \cdot \frac{Q}{Q_2} \cdot \frac{Q_2}{Q} \cdot \frac{\delta D_2}{\delta P} \right]} - 1$$

Therefore,

$$(5.26a) \quad \frac{\delta R_1}{\delta A_1} = \frac{\alpha_1}{e - (n_1 s_1 + n_2 s_2)} - 1$$

A29. PROOF TO EQUATION (5.29b)

$$(5.29b) \quad \alpha_1 = \frac{PQ}{A_1} \cdot s_1 \cdot \beta_1$$

Recall equation (5.29a)

$$(5.29a) \quad \alpha_1 = P \cdot \frac{\delta D_1}{\delta A_1}$$

Multiply the right-hand side of equation (5.27a)

$$\text{by } \frac{A_1}{A_1} \cdot \frac{Q_1}{Q_1}$$

This gives

$$\begin{aligned}\alpha_1 &= \frac{PQ_1}{A_1} \cdot \frac{A_1}{Q_1} \cdot \frac{\delta D_1}{\delta A_1} \\ &= \left[\frac{PQ_1}{A_1} \cdot \frac{Q}{Q_1} \right] \cdot \left[\frac{Q_1}{Q} \right] \cdot \left[\frac{A_1}{Q_1} \cdot \frac{\delta D_1}{\delta A_1} \right]\end{aligned}$$

That is,

$$(5.29b) \quad \alpha_1 = \frac{PQ}{A_1} \cdot s_1 \cdot \beta_1$$

A30. PROOF TO EQUATION (5.30a)

$$(5.30a) \quad \theta_1 = \frac{\beta_1 s_1}{e - (n_1 s_1 + n_2 s_2)}$$

Recall equation (5.27a).

$$(5.27a) \quad \frac{\alpha_1}{e - (n_1 s_1 + n_2 s_2)} = 1$$

Substitute (5.29b) into (5.27a)

Therefore,

$$\frac{\frac{PQ}{A_1} \cdot s_1 \cdot \beta_1}{e - (n_1 s_1 + n_2 s_2)} = 1$$

and

$$\frac{A_1}{PQ} = \frac{s_1 \beta_1}{e - (n_1 s_1 + n_2 s_2)}$$

That is,

$$(5.30a) \quad \theta_1 = \frac{\beta_1 s_1}{e - (n_1 s_1 + n_2 s_2)}$$

A31. PROOF TO EQUATION (5.32)

$$(5.32) \quad n_a = \sum_{i=1}^n n_i s_i$$

Take a two-market case. Then

$$Q = Q_1 + Q_2$$

$$\text{and} \quad \frac{\delta Q}{\delta P} = \frac{\delta Q_1}{\delta P} + \frac{\delta Q_2}{\delta P}$$

That is,

$$\frac{\delta Q}{\delta P} \cdot \frac{P}{Q} = \frac{\delta Q_1}{\delta P} \cdot \frac{P}{Q_1} \cdot \frac{Q_1}{Q} + \frac{\delta Q_2}{\delta P} \cdot \frac{P}{Q_2} \cdot \frac{Q_2}{Q}$$

That is,

$$n_a = n_1 s_1 + n_2 s_2$$

$$= \sum_{i=1}^2 n_i s_i$$

Therefore, for an n-market case

$$(5.32) \quad n_a = \sum_{i=1}^n n_i s_i$$

A32. DERIVATION OF EQUATION (5.31d) FROM (5.31c)

$$(5.31d) \quad \theta = \frac{\beta}{e + |n_a|}$$

Recall (5.31c)

$$(5.31c) \quad \theta = \frac{\sum_{i=1}^n \beta_i s_i}{e + |n_a|}$$

Let all $\beta_i = \beta$.

Therefore, (5.31c) becomes

$$\theta = \frac{\beta \sum_{i=1}^n s_i}{e + |n_a|}$$

That is,

$$(5.31d) \quad \theta = \frac{\beta}{e + |n_a|}$$

A33. PROOF TO EQUATION (5.33b)

$$(5.33b) \quad \Gamma_i = \frac{\beta_i s_i}{\sum_{i=1}^n \beta_i s_i}$$

Recall equation (5.30c) and equation (5.31c)

$$(5.30c) \quad \theta_i = \frac{\beta_i s_i}{e + |n_a|}$$

$$(5.31c) \quad \theta = \frac{\sum_{i=1}^n \beta_i s_i}{e + |n_a|}$$

Now note that by manipulating equation (5.33a)

$$(A32.1) \quad \Gamma_i = \frac{\theta_i}{\theta}$$

Therefore, by dividing equation (5.30c) by equation (5.31c)

$$(5.33b) \quad \Gamma_i = \frac{\beta_i s_i}{\sum_{i=1}^n \beta_i s_i}$$

A34. PROOF TO EQUATION (5.34b)

$$(5.34b) \quad \theta_i' = \frac{\beta_i}{e - n_a}$$

Recall equation (5.30c)

$$(5.30c) \quad \theta_i = \frac{\beta_i s_i}{e + |n_a|}$$

$$\text{That is } \theta_i = \frac{A_i}{PQ} = \frac{Q_i}{Q} \cdot \frac{\beta_i}{e + |n_a|}$$

Multiply both sides by $\frac{Q}{Q_i}$

That is

$$\frac{A_i}{PQ} \cdot \frac{Q}{Q_i} = \frac{Q}{Q_i} \cdot \frac{Q_i}{Q} \cdot \frac{\beta_i}{e + |n_a|}$$

Therefore,

$$(5.34b) \quad \theta_i' = \frac{\beta_i}{e - n_a}$$

A35. DERIVATION OF EQUATION (5.51b) FROM (5.51a)

$$(5.51b) \quad P_{a,p} = a_a - b_a Q_a + b_a V_a$$

$$\text{where } V_a = \sum_{i=1}^n V_i$$

Take (5.51a)

$$(5.51a) \quad P_{i,p} = a_i - b_i Q_i + b_i V_i \\ = a_i' - b_i Q_i$$

$$\text{where } a_i' = a_i + b_i V_i$$

Now, from proof A1 in Appendix 1,

$$(A35.1) \quad P_{a,p} = a_a' - b_a Q_a$$

where, for a two-market example,

$$\begin{aligned}
 a_a' &= \frac{a_1' b_2 + a_2' b_1}{b_1 + b_2} \\
 &= \frac{(a_1 + b_1 V_1) b_2 + (a_2 + b_2 V_2) b_1}{b_1 + b_2} \\
 &= \frac{a_1 b_2 + a_2 b_1}{b_1 + b_2} + \frac{b_1 b_2}{b_1 + b_2} (V_1 + V_2) \\
 &= a_a + b_a V_a
 \end{aligned}$$

Therefore,

$$(5.51) \quad P_{a,p} = a_a - b_a Q_a + b_a V_a$$

$$\text{where } V_a = \sum_{i=1}^n V_i$$

A36. PROOFS TO EQUATIONS (5.53a) AND (5.53b)

See A2, Appendix 1 for analagous proofs to equations (4.5a) and (4.5b).

A37. PROOF TO EQUATION (5.55b)

$$(5.55b) \quad \Delta TR = \frac{b_a V_a}{b_a + d} \left[2a_a - c - 2b_a \left[\frac{a_a - c}{b_a + d} \right] + b_a V_a \left[1 - \frac{b_a}{b_a + d} \right] \right]$$

Recall (5.55a)

$$(5.55a) \quad \Delta TR = P_{o,p} Q_{o,p} - P_o Q_o$$

Substituting (5.53a), (5.53b), (4.5a) and (4.5b) into (5.55a) gives

$$\Delta TR = \left[a_a - b_a \left[\frac{a_a - c + b_a V_a}{b_a + d} \right] + b_a V_a \right] \left[\frac{a_a - c + b_a V_a}{b_a + d} \right]$$

$$- \left[a_a - b_a \left[\frac{a_a - c}{b_a + d} \right] \right] \left[\frac{a_a - c}{b_a + d} \right]$$

Take the first two terms

$$\begin{aligned} & \frac{a_a(a_a - c + b_a V_a)}{b_a + d} - \frac{b_a(a_a - c + b_a V_a)^2}{(b_a + d)^2} + \frac{b_a V_a(a_a - c + b_a V_a)}{b_a + d} \\ &= \frac{a_a(a_a - c)}{b_a + d} + \frac{a_a b_a V_a}{b_a + d} - \frac{b_a(a_a - c)^2}{(b_a + d)^2} - \frac{2b_a^2 V_a(a_a - c)}{(b_a + d)^2} \\ & \quad - \frac{b_a^3 V_a^2}{(b_a + d)^2} + \frac{b_a V_a(a_a - c)}{b_a + d} + \frac{b_a^2 V_a^2}{b_a + d} \end{aligned}$$

Take the last two terms

$$\begin{aligned} & - \left[\frac{a_a(a_a - c)}{b_a + d} - \frac{b_a(a_a - c)^2}{(b_a + d)^2} \right] \\ &= - \frac{a_a(a_a - c)}{b_a + d} + \frac{b_a(a_a - c)^2}{(b_a + d)^2} \end{aligned}$$

Recombining all terms gives

$$\Delta TR = \frac{a_a b_a V_a + b_a V_a(a_a - c) + b_a^2 V_a^2}{b_a + d} - \frac{2b_a^2 V_a(a_a - c) + b_a^3 V_a^2}{(b_a + d)^2}$$

Therefore,

$$(5.55b) \quad \Delta TR = \frac{b_a V_a}{b_a + d} \left[2a_a - c - 2b_a \left[\frac{a_a - c}{b_a + d} \right] + b_a V_a \left[1 - \frac{b_a V_a}{b_a + d} \right] \right]$$

A38. PROOF TO EQUATION (5.56c)

$$(5.56c) \quad \Delta C = \frac{b_a V_a}{b_a + d} \left[c + \frac{d(a_a - c)}{b_a + d} + b_a V_a \frac{d}{2(b_a + d)} \right]$$

Recall (5.56b)

$$(5.56b) \quad \Delta C = c(Q_{op} - Q_o) + \frac{d}{2} (Q_{op}^2 - Q_o^2)$$

By substituting (5.53b) and (4.5b) into (5.56b)

$$\begin{aligned} \Delta C &= c \left[\frac{a_a - c + b_a V_a}{b_a + d} - \frac{a_a - c}{b_a + d} \right] + \frac{d}{2} \left[\left[\frac{a_a - c + b_a V_a}{b_a + d} \right]^2 - \left[\frac{a_a - c}{b_a + d} \right]^2 \right] \\ &= c \left[\frac{b_a V_a}{b_a + d} \right] + \frac{d}{2} \left[\frac{b_a V_a (2a_a - 2c + b_a V_a)}{(b_a + d)^2} \right] \\ &= \frac{b_a V_a}{b_a + d} \left[c + \frac{d}{2} \left[\frac{2a_a - 2c - b_a V_a}{b_a + d} \right] \right] \end{aligned}$$

That is,

$$(5.56c) \quad \Delta C = \frac{b_a V_a}{b_a + d} \left[c + \frac{d(a_a - c)}{b_a + d} + b_a V_a \frac{d}{2(b_a + d)} \right]$$

A39. PROOF TO EQUATION (5.57)

$$\begin{aligned} (5.57) \quad \Delta PS &= \frac{b_a V_a}{b_a + d} \left[2(a_a - c) - \left[\frac{a_a - c}{b_a + d} \right] (2b_a + d) \right. \\ &\quad \left. + b_a V_a \left[\frac{d}{2(b_a + d)} \right] \right] \end{aligned}$$

Subtract equation (5.56b) from (5.55b)

$$\begin{aligned} \Delta PS &= \frac{b_a V_a}{b_a + d} \left[2a_a - c - 2b_a \left[\frac{a_a - c}{b_a + d} \right] + b_a V_a \left[1 - \frac{b_a}{b_a + d} \right] \right] \\ &\quad - \frac{b_a V_a}{b_a + d} \left[c + \frac{d(a_a - c)}{b_a + d} + b_a V_a \frac{d}{2(b_a + d)} \right] \end{aligned}$$

$$\begin{aligned}
&= \frac{b_a V_a}{b_a + d} \left[2a_a - 2c - \left[\frac{a_a - c}{b_a + d} \right] [2b_a + d] + b_a V_a \left[1 - \frac{b_a}{b_a + d} - \frac{d}{2(b_a + d)} \right] \right] \\
&= \frac{b_a V_a}{b_a + d} \left[2a_a - 2c - \left[\frac{a_a - c}{b_a + d} \right] [2b_a + d] + b_a V_a \left[1 - \frac{2b_a + d}{2(b_a + d)} \right] \right] \\
&= \frac{b_a V_a}{b_a + d} \left[2(a_a - c) - \left[\frac{a_a - c}{b_a + d} \right] [2b_a + d] + b_a V_a \left[\frac{2b_a + 2d - 2b_a - d}{2(b_a + d)} \right] \right]
\end{aligned}$$

Therefore,

$$(5.57) \Delta PS = \frac{b_a V_a}{b_a + d} \left[2[a_a - c] - \left[\frac{a_a - c}{b_a + d} \right] [2b_a + d] + b_a V_a \left[\frac{d}{2(b_a + d)} \right] \right]$$

A40. PROOF TO EQUATION (5.58)

From Section 4.4.2 of Chapter 4

$$(4.21) P_{ad} = a_a - b_a Q_a + \frac{k}{Q_a}$$

Therefore, by analogy,

$$(5.58) P_{adp} = a_a' - b_a Q_a + \frac{k'}{Q_a}$$

where $a_a' = a_a + b_a V_a$

$$\text{and } k' = \sum_{i=1}^n \left[\frac{\{(a_i - a_a) + (b_i V_i - b_a V_a)\}^2}{4b_i} \right]$$

$$\text{with } V_a = \sum_{i=1}^n V_i$$

From A35, Appendix 2,

$$a_a' = a_a + b_a V_a$$

$$\text{and } V_a = \sum_{i=1}^n V_i$$

From A10, Appendix 1,

$$k = \sum_{i=1}^n \left[\frac{(a_i - a_a)^2}{4b_i} \right]$$

$$\text{Therefore } k' = \sum_{i=1}^n \left[\frac{(a_i' - a_a')^2}{4b_i} \right]$$

$$= \sum_{i=1}^n \left[\frac{\{(a_i + b_i V_i) - (a_a + b_a V_a)\}^2}{4b_i} \right]$$

$$= \sum_{i=1}^n \left[\frac{\{(a_i - a_a) + (b_i V_i - b_a V_a)\}^2}{4b_i} \right]$$

A41. PROOF TO EQUATIONS (5.60a) and (5.60b)

For (5.60b), see A11 (Appendix 1), for an analogous proof to equation (4.24a).

For (5.60a), substitute (5.60b) into the supply curve, equation (4.3).

A42. PROOF TO EQUATION (4.23)

Substitute (4.24a) into the supply curve, equation (4.3).

A43. PROOF TO EQUATION (5.61b)

$$\begin{aligned}
 (5.61b) \quad \Delta TR &= \frac{c(a_a' - a_a) + c(X'^{0.5} - X^{0.5}) + 2d(k' - k)}{2(b_a + d)} \\
 &+ \frac{2d\{[(a_a'^2 - a_a^2) - 2c(a_a' - a_a)]c + d\}}{4(b_a + d)^2} \\
 &+ \frac{[(a'X'^{0.5} - aX^{0.5}) - c(X'^{0.5} - X^{0.5})]}{4(b_a + d)^2}
 \end{aligned}$$

Recall (5.61a)

$$(5.61a) \quad \Delta TR = P_{d1p}Q_{1p} - P_{d1}Q_1$$

Take the first term. From (5.60a) and (5.60b)

$$\begin{aligned}
 P_{d1p}Q_{1p} &= \left[\frac{a_a' - c + X'^{0.5}}{2(b_a + d)} \right] \left[c + d \frac{a_a' - c + X'^{0.5}}{2(b_a + d)} \right] \\
 &= \frac{c(a_a' - c + X'^{0.5})}{2(b_a + d)} + \frac{d(a_a' - c + X'^{0.5})^2}{4(b_a + d)^2} \\
 &= \frac{c(a_a' - c)}{2(b_a + d)} + \frac{cX'^{0.5}}{2(b_a + d)} + \frac{d[(a_a' - c) + X'^{0.5}][(a_a' - c) + X'^{0.5}]}{4(b_a + d)^2} \\
 &= \frac{c(a_a' - c)}{2(b_a + d)} + \frac{cX'^{0.5}}{2(b_a + d)} + \frac{d[(a_a' - c)^2 + 2(a_a' - c)X'^{0.5} + X']}{4(b_a + d)^2} \\
 &= \frac{c(a_a' - c)}{2(b_a + d)} + \frac{cX'^{0.5}}{2(b_a + d)} + \frac{d(a_a' - c)^2}{4(b_a + d)^2} + \frac{2dX'^{0.5}(a_a' - c)}{4(b_a + d)^2} \\
 &+ \frac{dX'}{4(b_a + d)^2}
 \end{aligned}$$

Take the final term of this expression

$$\begin{aligned}
 \frac{dX'}{4(b_a + d)^2} &= \frac{d[(a_a' - c)^2 + 4k'(b_a + d)]}{4(b_a + d)^2} \\
 &= \frac{d(a_a' - c)^2}{4(b_a + d)^2} + \frac{dk'}{b_a + d}
 \end{aligned}$$

Therefore,

$$P_{d1p}Q_{1p} = \frac{c(a_a' - c)}{2(b_a + d)} + \frac{cX'^{0.5}}{2(b_a + d)} + \frac{2d(a_a' - c)^2}{4(b_a + d)^2} + \frac{2dX'^{0.5}(a_a' - c)}{4(b_a + d)^2} + \frac{dk'}{(b_a + d)}$$

Take $P_{d1}Q_1$. By analagous reasoning

$$P_{d1}Q_1 = \frac{c(a_a - c)}{2(b_a + d)} + \frac{cX^{0.5}}{2(b_a + d)} + \frac{2d(a_a - c)^2}{4(b_a + d)^2} + \frac{2dX^{0.5}(a_a - c)}{4(b_a + d)^2} + \frac{dk}{(b_a + d)}$$

Therefore,

$$\begin{aligned} \Delta TR = & \left[\frac{c(a_a' - c) - c(a_a - c)}{2(b_a + d)} \right] + \left[\frac{cX'^{0.5} - cX^{0.5}}{2(b_a + d)} \right] \\ & + \left[\frac{2d(a_a' - c)^2 - 2d(a_a - c)^2}{4(b_a + d)^2} \right] \\ & + \left[\frac{2dX'^{0.5}(a_a' - c) - 2dX^{0.5}(a_a - c)}{4(b_a + d)^2} \right] + \left[\frac{dk' - dk}{(b_a + d)} \right] \end{aligned}$$

Take term 1 of this expression

$$\frac{c(a_a' - c) - c(a_a - c)}{2(b_a + d)} = \frac{c(a_a' - a_a)}{2(b_a + d)}$$

Take term 2 of this expression

$$\frac{cX'^{0.5} - cX^{0.5}}{2(b_a + d)} = \frac{c(X'^{0.5} - X^{0.5})}{2(b_a + d)}$$

Take term 3 of this expression

$$\frac{2d(a_a' - c)^2 - 2d(a_a - c)^2}{4(b_a + d)^2}$$

$$= \frac{2d(a_a'^2 - 2a_a'c + c^2 - a_a^2 + 2a_ac - c^2)}{4(b_a + d)^2}$$

$$= \frac{2d([a_a'^2 - a_a^2] - 2c[a_a' - a_a])}{4(b_a + d)^2}$$

Take term 4 of this expression

$$\begin{aligned} & \frac{2dX'^{0.5}(a_a' - c) - 2dX^{0.5}(a_a - c)}{4(b_a + d)^2} \\ &= \frac{2d(a'X'^{0.5} - cX'^{0.5} - a_aX^{0.5} + cX^{0.5})}{4(b_a + d)^2} \\ &= \frac{2d[a'X'^{0.5} - a_aX^{0.5}] - c[X'^{0.5} - X^{0.5}]}{4(b_a + d)^2} \end{aligned}$$

Take term 5 of this expression

$$\begin{aligned} & \frac{dk' - dk}{(b_a + d)} \\ &= \frac{d(k' - k)}{(b_a + d)} \end{aligned}$$

Therefore, putting all terms back together again gives

$$\begin{aligned} (5.61b) \Delta TR &= \frac{c(a_a' - a_a) + c(X'^{0.5} - X^{0.5}) + 2d(k' - k)}{2(b_a + d)} \\ &+ \frac{2d\{[(a_a'^2 - a_a^2) - 2c(a_a' - a_a)] + [(a_a'X'^{0.5} - a_aX^{0.5}) - c(X'^{0.5} - X^{0.5})]\}}{4(b_a + d)^2} \end{aligned}$$

A44. PROOF TO EQUATION (5.62b)

$$(5.62b) \Delta C = \frac{c(a_a' - a_a) + c(X'^{0.5} - X^{0.5}) + d(k' - k)}{2(b_a + d)}$$

$$+ \frac{d[(a_a'^2 - a_a^2) - 2c(a_a' - a_a) + (a_a' X'^{0.5} - a_a X^{0.5}) - c(X'^{0.5} - X^{0.5})]}{4(b_a + d)^2}$$

Recall (5.62a)

$$(5.62a) \quad \Delta C = c(Q_{1p} - Q_1) + \frac{d}{2} (Q_{1p}^2 - Q_1^2)$$

Substitute (5.60b) and (4.24a) into (5.62a), and take term 1 of the expression above.

$$\begin{aligned} c(Q_{1p} - Q_1) &= c \left[\frac{[(a_a' - c) + X'^{0.5}] - [(a_a - c) + X^{0.5}]}{2(b_a + d)} \right] \\ &= \frac{c[(a_a' - a_a) + (X'^{0.5} - X^{0.5})]}{2(b_a + d)} \\ &= \frac{c(a_a' - a_a)}{2(b_a + d)} + \frac{c(X'^{0.5} - X^{0.5})}{2(b_a + d)} \end{aligned}$$

Take term 2 of the expression in (5.62a)

$$\begin{aligned} \frac{d}{2} (Q_{1p}^2 - Q_1^2) &= \frac{d}{2} \left[\left[\frac{a_a' - c + X'^{0.5}}{2(b_a + d)} \right]^2 - \left[\frac{a_a - c + X^{0.5}}{2(b_a + d)} \right]^2 \right] \\ &= \frac{d}{2} \left[\frac{a_a'^2 - a_a'c + a_a'X'^{0.5} - a_a'c + c^2 - cX'^{0.5} + a_a'X'^{0.5} - cX'^{0.5} + X'}{4(b_a + d)^2} \right. \\ &\quad \left. - \frac{a_a^2 - a_ac + a_aX^{0.5} - a_ac + c^2 - cX^{0.5} + a_aX^{0.5} - cX^{0.5} + X}{4(b_a + d)^2} \right] \\ &= \frac{d}{2} \left[\frac{a_a'^2 - 2a_a'c + c^2 + 2a_a'X'^{0.5} - 2cX'^{0.5} + X' - a_a^2 + 2a_ac - c^2 - cXd}{4(b_a + d)^2} \right] \end{aligned}$$

$$\left[\frac{-2a_a X^{0.5} + 2cX^{0.5} - X}{ } \right]$$

Take $(X' - X)$

$$\begin{aligned} X' - X &= (a_a' - c)^2 + 4k'(b_a + d) - (a_a - c)^2 - 4k(b_a + d) \\ &= a_a'^2 - 2a_a'c + c^2 - a_a^2 + 2a_ac - c^2 + 4(b_a + d)(k' - k) \\ &= (a_a'^2 - a_a^2) - 2c(a_a' - a_a) + 4(b_a + d)(k' - k) \end{aligned}$$

Substituting this back into term 2 and rearranging gives

$$\begin{aligned} &= \frac{d}{2} \left[\frac{2(a_a'^2 - a_a^2) - 4c(a_a' - a_a) + 2(a_a' X'^{0.5} - a_a X^{0.5})}{4(b_a + d)^2} \right] ctd \\ &\quad - \left[\frac{2c(X'^{0.5} - X^{0.5})}{ } \right] + \frac{d(k' - k)}{2(b_a + d)} \end{aligned}$$

Putting terms 1 and 2 back together gives

$$\begin{aligned} (5.62b) \quad \Delta C &= \frac{c(a_a' - a_a) + c(X'^{0.5} - X^{0.5}) + d(k' - k)}{2(b_a + d)} \\ &+ \frac{d[(a_a'^2 - a_a^2) - 2c(a_a' - a_a) + (a_a' X'^{0.5} - a_a X^{0.5}) - c(X'^{0.5} - X^{0.5})]}{4(b_a + d)^2} \end{aligned}$$

A45. PROOF TO EQUATION (5.63)

Subtracting (5.62b) from (5.61b) gives

$$\begin{aligned} \Delta PS &= \frac{c(a_a' - a_a) + c(X'^{0.5} - X^{0.5}) - c(a_a' - a_a) - c(X'^{0.5} - X^{0.5})}{2(b_a + d)} ctd \\ &\quad + \frac{2d(k' - k) - d(k' - k)}{ } \\ &+ \frac{2d[(a_a'^2 - a_a^2) - 2c(a_a' - a_a)] - d[(a_a'^2 - a_a^2) - 2c(a_a' - a_a)]}{4(b_a + d)^2} ctd \end{aligned}$$

$$\begin{aligned}
& + \frac{d[(a'_a X'^{0.5} - a X^{0.5}) - c(X'^{0.5} - X^{0.5})] - d[(a'_a X'^{0.5} - a X^{0.5}) c t d}{- c(X'^{0.5} - X^{0.5})]} \\
& = \frac{d(k' - k)}{2(b_a + d)} + \frac{d\{[(a'_a{}^2 - a_a{}^2) - 2c(a'_a - a_a)] + c t d}{4(b_a + d)^2} \\
& \quad \frac{[(a'_a X'^{0.5} - a X^{0.5}) - c(X'^{0.5} - X^{0.5})]}{
\end{aligned}$$

Therefore,

$$\begin{aligned}
\Delta PS = & \frac{d\{[2(b_a + d)(k' - k)] + [(a'_a{}^2 - a_a{}^2) - 2c(a'_a - a_a)] c t d}{4(b_a + d)^2} \\
& + \frac{[(a'_a X'^{0.5} - a X^{0.5}) - c(X'^{0.5} - X^{0.5})]}{
\end{aligned}$$

APPENDIX 3

PROOFS TO SELECTED EQUATIONS IN CHAPTER 6

A46. PROOF TO EQUATION (6.5)

$$(6.5) \quad \Delta TR = k \quad \text{when } e = 0$$

Recall equation (4.32d)

$$(4.32d) \quad \Delta TR = k + \left[\frac{P_0^2 Q_0^2}{4} + P_0 Q_0 k \frac{n_a e}{n_a + e} \right]^{0.5} - \frac{P_0 Q_0}{2} - k \frac{e}{n_a + e}$$

As $e \rightarrow 0$, $(n_a + e) \rightarrow n_a$ and

$$\Delta TR \rightarrow k + \left[\frac{P_0^2 Q_0^2}{4} \right]^{0.5} - \frac{P_0 Q_0}{2}$$

Therefore,

$$(6.5) \quad \Delta TR = k \quad \text{when } e = 0.$$

A47. PROOF TO EQUATION (6.6)

$$(6.6) \quad \Delta TR = \left[\frac{P_0^2 Q_0^2}{4} + P_0 Q_0 k n_a \right]^{0.5} - \frac{P_0 Q_0}{2} \quad \text{when } e \rightarrow \infty$$

Recall equation (4.32d)

$$(4.32d) \quad \Delta TR = k + \left[\frac{P_0^2 Q_0^2}{4} + P_0 Q_0 k \frac{n_a e}{n_a + e} \right]^{0.5} - \frac{P_0 Q_0}{2} - k \frac{e}{n_a + e}$$

Let $x = n_a + e$

Therefore,

$$e = x - n_a$$

Therefore,

$$\begin{aligned}\Delta TR &= k + \left[\frac{P_0^2 Q_0^2}{4} + P_0 Q_0 k n_a \left[1 - \frac{n_a}{x} \right] \right]^{0.5} - \frac{P_0 Q_0}{2} - k \left[\frac{x - n_a}{x} \right] \\ &= k + \left[\frac{P_0^2 Q_0^2}{4} + P_0 Q_0 k n_a \left[1 - \frac{n_a}{x} \right] \right]^{0.5} - \frac{P_0 Q_0}{2} - k \left[1 - \frac{n_a}{x} \right]\end{aligned}$$

Now, as $e \rightarrow \infty$, $x \rightarrow \infty$ and $\frac{n_a}{x} \rightarrow 0$.

Therefore, as $e \rightarrow \infty$

$$\Delta TR \rightarrow k + \left[\frac{P_0^2 Q_0^2}{4} + P_0 Q_0 k n_a \right]^{0.5} - \frac{P_0 Q_0}{2} - k$$

That is,

$$(6.6) \quad \Delta TR = \left[\frac{P_0^2 Q_0^2}{4} + P_0 Q_0 k n_a \right]^{0.5} - \frac{P_0 Q_0}{2} \quad \text{when } e \rightarrow \infty$$

APPENDIX 4

Reprint of

Martin, S. K., Young, L, and Zwart, A. C.

Optimal Pricing and Promotion for Agricultural
Marketing Agencies. Research Report No. 177.

Agricultural Economics Research Unit, Lincoln College.

1986. 21 p.

**OPTIMAL PRICING AND PROMOTION
FOR AGRICULTURAL MARKETING AGENCIES**

S.K. Martin
L. Young
A.C. Zwart

RESEARCH REPORT No. 177

February 1986

**Agricultural Economics Research Unit
Lincoln College
Canterbury
New Zealand**

ISSN 0069-3790

CONTENTS

	Page
PREFACE	(ii)
CHAPTER 1 INTRODUCTION	1
CHAPTER 2 ALTERNATIVE MODELS AND THEIR OPTIMAL MARKETING STRATEGIES	3
CHAPTER 3 THEORETICAL ANALYSIS	11
CHAPTER 4 CONCLUSIONS	19

PREFACE

Marketing agencies which operate in international markets are faced with pricing and promotion decisions in individual markets. Economic theory can assist in these tasks by providing a framework for evaluating specific commercial strategies in particular products.

Therefore, the Agricultural Economics Research Unit has an interest in theoretical research which widens our understanding of optimal decision-making by agencies operating in agricultural markets.

Mrs S.K. Martin has been considering aspects of the economics of market segmentation by agricultural marketing institutions as part of her doctoral dissertation under the supervision of Professor A.C. Zwart. This Research Report outlines one aspect of this research. It was undertaken in collaboration with Professor L. Young from the University of Texas, Austin, Texas, USA. Professor Young derived the solutions to the problem established in Chapter 3.

R.G. Lattimore
Director

1

CHAPTER 1

INTRODUCTION

In recent years, marketing institutions operating in New Zealand's agricultural export sector have placed increasing emphasis on market segmentation strategies as an economic instrument. Indications that marketing institutions have moved towards this type of policy are evident in attempts to diversify markets, and to gear promotion activities and product development to specific market segments. The change in policy emphasis by these agencies reflects the increasing influence of the prescriptions of marketing management in agricultural marketing.

When the general term 'market segmentation' is used in the marketing management context, it tends to refer to the practices of segmenting a market, targeting specific market segments, and positioning products within these segments. Product positioning requires the development of a marketing mix for each target segment using a particular blend of controllable marketing variables (Kotler, 1984).

In attempting to apply these principles in the markets for agricultural products, agencies are faced with the problem of how much product to allocate to individual market segments, and what pricing and promotion strategies to adopt in each of these segments. The prescriptions of economic theory can assist in these tasks, by indicating optimal strategies for a particular marketing agency objective. In the literature, attention has been directed towards this problem of determining optimal marketing mixes (Lambin, 1976). However, much of it uses extensions to the theory of monopolistic or oligopolistic markets, but in typical

agricultural industries these market conditions do not apply.

In this Report, the development of economic models of marketing behaviour are discussed, and the analysis is extended to consider the specific environment faced by an agricultural marketing agency. The major feature of such analysis is the incorporation of a competitive supply response in a model which determines the optimal pricing and promotion strategies in more than one market segment.

The following Chapter describes the development of such models, while Chapter 3 focusses on their extension. The final Chapter compares the alternative model prescriptions for pricing and promotion.

CHAPTER 2

ALTERNATIVE MODELS AND THEIR
OPTIMAL MARKETING STRATEGIES

Since a great deal of attention in the literature has been directed towards the marketing behaviour of a monopolistic firm (Lambin, 1976), a generalised version of this problem will be discussed, and variants and extensions of this general model will be subsequently examined.

Consider a monopolist who operates in a number of predetermined market segments, and provides a product of identical quality to each of these segments. In this case, demand in the i th market segment, Q_i , can be written as

$$(1) \quad Q_i = Q_i(P_i, A_i)$$

where P_i and A_i are price and advertising, respectively, in that segment. Aggregate demand, Q , is then given by

$$(2) \quad Q = \sum_i Q_i(P_i, A_i)$$

If the firm maximises profit net of advertising costs, then its profit function, Π , is

$$(3) \quad \Pi = \sum_i P_i Q_i(P_i, A_i) - C(Q) - \sum_i A_i$$

where $C(Q)$ is the total cost of producing output Q . To develop appropriate decision rules for marketing mix optimisation, this objective function, Π , would be maximised.

Variants of this generalised problem have been examined in the literature. For example, Dorfman and Steiner (1954) considered marketing mix optimisation by a monopolist in one (aggregate) market, where the decision variables available to the firm are price or output, and advertising. In this case, demand, Q , is given by

$$(4) \quad Q = Q(P, A)$$

and the profit function, Π , by

$$(5) \quad \Pi = PQ(P, A) - C(Q) - A$$

where P and A are price and advertising.

When this profit function is maximised, it yields the optimal advertising decision rule

$$(6) \quad \theta = \beta \left[\frac{P - MC}{P} \right]$$

where θ is the advertising to sales ratio, $A/(PQ)$, β is the advertising elasticity of demand, and MC is marginal cost. The corresponding product-price decision rule is given by

$$(7) \quad \left[\frac{P - MC}{P} \right] = 1/\eta$$

where η is the price elasticity of demand (absolute value). This is the familiar profit-maximising rule where marginal cost equals marginal revenue.

These optimal product-price and advertising decision rules can be expressed in a single relationship which encapsulates both rules as follows.

$$(8) \quad \theta = \beta/\eta$$

Equation (8) has become known as the Dorfman-Steiner theorem, and is appropriate for a monopolist operating in a single market, which includes a marketing agency concerned with aggregate demand and with the ability to control output. This theorem of optimal advertising by a monopolist has been extended to include oligopolistic market structures (Lambin, 1976), and from its static formulation to include the dynamics of the sales response to advertising (Nerlove and Arrow, 1962).

Although the Dorfman-Steiner model considers price, product and promotion as elements in its marketing mix, it abstracts from the fourth variable of place, or market segments. An alternative model which does this, but which abstracts from promotion, is the familiar model of monopolistic price discrimination. In this case, demand, Q , is given by

$$(9) \quad Q = \sum_i Q_i(P_i)$$

and the profit function, Π by

$$(10) \quad \Pi = \sum_i P_i Q_i(P_i) - C(Q)$$

Maximisation of this profit function gives the familiar output and pricing rules for a price discriminating monopolist. That is,

$$(11) \quad MR_1 = MR_2 = \dots = MR_i = \dots = MC$$

where MR_i is marginal revenue in the i th market and MC is the marginal cost of production.

The decision rules derived from these models give partial indicators as to how a profit-maximising monopolist might optimally choose a marketing mix or mixes in specific

circumstances. However, such prescriptions are not appropriate for a marketing agency operating in a typically structured agricultural industry. When operating collectively on behalf of producers, such institutions may be able to exert monopoly power in their markets. However, in New Zealand, they do not have the power to restrict output by producers. Therefore, when producers receive higher returns, in the form of a pool price, which results from demand management strategies in individual market segments, they may respond by increasing output accordingly. Unlike the monopoly case, where output is a decision variable which can be optimised, output is determined competitively.

Nerlove and Waugh (1961) recognised these supply-side differences between a monopolist and a typical agricultural marketing agency. Assuming the above agricultural supply conditions, they considered the optimal advertising decision for such agencies operating in one (aggregate) market. In their model, demand, Q , is given by

$$(12) \quad Q = Q(P, A)$$

and supply can be represented by

$$(13) \quad S = S(P)$$

where S is the output supplied at price P . The profit function to be maximised then becomes

$$(14) \quad \Pi = PQ(P, A) - C(S(P)) - A$$

where $C(S)$, the aggregate cost of production, is the area under the supply curve to the left of S . However, this profit function must be maximised subject to the constraint that excess supply is zero. That is,

$$(15) \quad Q(P, A) = S(P)$$

The solution to the Nerlove-Waugh model yields the following advertising decision rule

$$(16) \quad \theta = \frac{\beta}{\eta + \epsilon}$$

where ϵ is the price elasticity of supply and other variables are as defined for the Dorfman-Steiner model.

In the Nerlove-Waugh case, the optimal promotion decision can be determined by the marketing agency, whereas the product-price decision is determined by the market. However, like its counterpart, the Dorfman-Steiner theorem, this model abstracts from the marketing mix variable, place, since it does not consider pricing and promotion strategies in individual market segments.

An attempt was made by De Boer (1977) to examine the direction of advertising effort to individual market segments under discriminatory pricing between these segments. However, his prescriptions for agricultural marketing agencies are not necessarily valid, since he assumes monopolistic supply features. In fact, the theory of advertising under competitive agricultural supply conditions has advanced little since the Nerlove-Waugh theorem (Strak, 1985).

An obvious extension to the Nerlove-Waugh theorem would be to consider the allocation of optimal advertising effort among a number of market segments, where price in each of these segments is determined by aggregate (total) demand and supply conditions. This has been done by Martin (1985).

In this case, demand in market segment i is given by

$$(17) \quad Q_i = Q_i(P, A_i)$$

and aggregate demand by

$$(18) \quad Q = \sum_i Q_i(P, A_i)$$

Supply is represented by

$$(19) \quad S = S(P)$$

The profit function to be maximised is given by

$$(20) \quad \Pi = P \sum_i Q_i(P, A_i) - C(S(P)) - \sum_i A_i$$

subject to the constraint that

$$(21) \quad \sum_i Q_i(P, A_i) = S(P)$$

In this case, optimal advertising effort in market segment i , is given by

$$(22) \quad \theta_i = \frac{\beta_i}{\eta + \varepsilon}$$

where $\theta_i = \frac{A_i}{PQ_i}$ and β_i is the advertising elasticity of

demand in market segment i , with all other variables being defined as for the Nerlove-Waugh model.

In a two market segment case, the relative direction of advertising effort can be given by the following ratio.

$$(23) \quad \frac{A_1/Q_1}{A_2/Q_2} = \frac{\beta_1}{\beta_2}$$

That is, the ratio of advertising per unit sales in one market segment to that in the other market segment is equal to the ratio of the corresponding advertising elasticities.

Although the above model extends the Nerlove-Waugh theorem to consider a number of market segments, it abstracts from optimal pricing policies which such an institution might pursue when it has the power to control the allocation of industry output among alternative market segments. Consequently, the next Chapter develops a marketing mix optimisation model which yields decision rules for optimal pricing and promotion in individual market segments, and which takes account of typical agricultural supply features.

In such a case, demand is represented by

$$(24) \quad Q = \sum_i Q_i(P_i, A_i)$$

and supply by

$$(25) \quad S = S(R)$$

where R is the return per unit of output, or pool price, received by the producer. This return is given by

$$(26) \quad R = \frac{\sum_i P_i Q_i(P_i, A_i)}{\sum_i Q_i(P_i, A_i)}$$

The profit function to be maximised is

$$(27) \quad \Pi = \sum_i P_i Q_i(P_i, A_i) - C(S(R)) - \sum_i A_i$$

where $C(S)$ is defined as for the Nerlove-Waugh model.

As with the Nerlove-Waugh case, the marketing agency is constrained to adopt policies such that it sells all the output supplied when producers receive the average return, R . That is,

$$(28) \quad \sum_i Q_i(P_i, A_i) = S(R)$$

The solution to this constrained maximisation problem would yield decision rules for the optimal allocation of output, and therefore prices, in individual market segments, and for the optimal allocation of advertising effort to these segments. However, the aggregate output produced is determined by market forces.

CHAPTER 3

THEORETICAL ANALYSIS

For analytical ease, a two-market segment case of the generalised model outlined in the previous Chapter will be considered. To avoid cumbersome mathematical expressions, some of the notation will also be redefined.

Let $d(p,a)$ be the demand in the first market segment where the price is p and advertising expenditure is a . The corresponding variables for the second market segment are denoted by the corresponding upper case letters. Producers receive the average return per unit of output, or the pool price,

$$(29) \quad r(p,P,a,A) \equiv \frac{pd(p,a)}{d(p,a)} + \frac{PD(P,A)}{D(P,A)}$$

Let the supply at this pool price be $s(r)$. The aggregate cost of production of supply, s , is the area, $c(s)$, under the supply curve to the left of s . The marketing agency maximises aggregate profits net of advertising costs.

$$(30) \quad \Pi(p,P,a,A) \equiv pd(p,a) + PD(P,A) - c(d(p,a) + D(P,A)) - a - A$$

Let excess supply be

$$(31) \quad x(p,P,a,A) \equiv s(r(p,P,a,A)) - d(p,a) - D(P,A)$$

The marketing agency is constrained to adopt policies such that it sells all output supplied when producers receive the pool price, r .

Therefore, it solves

$$(32) \quad \max_{p, P, a, A} \Pi(p, P, a, A) \text{ subject to } x(p, P, a, A) = 0$$

Let λ be the Lagrange multiplier associated with the constraint in (32). Using a subscript to denote partial differentiation with respect to the corresponding variable, the Lagrange equations can be written in the form

$$\frac{\Pi_p}{x_p} = \frac{\Pi_P}{x_P} = \frac{\Pi_a}{x_a} = \frac{\Pi_A}{x_A} = \lambda$$

or

$$(33) \quad \frac{d + (p - c_s)d_p}{s_r r_p - d_p} = \frac{D + (P - c_s)D_P}{s_r r_P - D_P} = \frac{(p - c_s)d_a - 1}{s_r r_a - d_a} = \frac{(P - c_s)D_A - 1}{s_r r_A - D_A}$$

Since $c(s)$ is the area under the supply curve to the left of s ,

$$(34) \quad c_s(s(r)) = r,$$

Moreover, recalling the definition (29) of r ,

$$(35) \quad p - r = p - (pd + PD)/(d + D) \doteq m/d$$

where

$$(36) \quad m \equiv (p - P)dD/(d + D)$$

Note that with this definition

$$(37) \quad P - r = -m/D$$

Using (34) - (37), the first order conditions (33) simplify to

$$(38) \quad \frac{d + m d_p / d}{s_r r_p - d_p} = \frac{D - m D_P / D}{s_r r_P - D_P} = \frac{m d_a / d - 1}{s_r r_a - d_a} = \frac{m D_A / D - 1}{s_r r_A - D_A}$$

or

$$(39) \quad \frac{\frac{p d + m p d_p / d}{s r_s \cdot \frac{p r}{r} - \frac{p d}{d}}}{\frac{p d + m p d_p / d}{s r_s \cdot \frac{p r}{r} - \frac{p d}{d}}} = \frac{\frac{P D - m P D_P / D}{s r_s \cdot \frac{P r}{r} - \frac{P D}{D}}}{\frac{P D - m P D_P / D}{s r_s \cdot \frac{P r}{r} - \frac{P D}{D}}} =$$

$$\frac{\frac{m a d_a - a}{s r_s \cdot \frac{a r}{r} - \frac{a d}{d}}}{\frac{m a d_a - a}{s r_s \cdot \frac{a r}{r} - \frac{a d}{d}}} = \frac{\frac{-m A D_A / D - A}{s r_s \cdot \frac{A r}{r} - \frac{A D}{D}}}{\frac{-m A D_A / D - A}{s r_s \cdot \frac{A r}{r} - \frac{A D}{D}}}$$

Define the elasticities

$$(40) \quad e^P \equiv -p d_p / d; \quad e^a \equiv a d_a / d; \quad t^P \equiv p r_p / r; \quad t^a \equiv a r_a / r; \quad f^r \equiv r s_r / s$$

with similar definitions for the second market segment using the corresponding upper case symbols. Then (40) becomes

$$(41) \quad \frac{\frac{p d}{s f^r t^P} - \frac{m e^P}{d e^P}}{\frac{p d}{s f^r t^P} + \frac{m e^P}{d e^P}} = \frac{\frac{P D}{s f^r T^P} - \frac{m E^P}{D E^P}}{\frac{P D}{s f^r T^P} + \frac{m E^P}{D E^P}} = \frac{\frac{m e^a}{s f^r t^a} - \frac{a}{e^a d}}{\frac{m e^a}{s f^r t^a} - \frac{a}{e^a d}} = \frac{\frac{-m E^A}{s f^r T^A} - \frac{A}{E^A D}}{\frac{-m E^A}{s f^r T^A} - \frac{A}{E^A D}}$$

The transmission elasticities are now evaluated.

Since

$$r \equiv \frac{p d(p, a)}{d(p, a)} + \frac{P D(P, A)}{D(P, A)}$$

then

$$\begin{aligned}
 r_a/r &= \frac{pdd_a + pDd_a - pdd_a - QDd_a}{(pd + QD)(d + D)} \\
 &= \frac{Dd_a(p - P)}{(pd + PD)(d + D)}
 \end{aligned}$$

Therefore

$$\begin{aligned}
 (42) \quad t^a &\equiv ar_a/r \\
 &= \frac{ad_a(p - P)D}{(pd + PD)(d + D)} \\
 &= \frac{e^a dD(p - P)}{(pd + QD)(d + D)} \\
 &= \frac{e^a_m}{(pd + PD)}
 \end{aligned}$$

Similarly

$$\begin{aligned}
 (43) \quad T^A &\equiv Ar_A/r \\
 &= \frac{E^A dD(P - p)}{(pd + PD)(d + D)} \\
 &= \frac{-E^A_m}{(pd + PD)}
 \end{aligned}$$

Also

$$r_p/r = \frac{d(d + D) + pd_d + pd_D - pd_d - PDd_p}{(pd + PD)(d + D)} - \frac{PDd_p}{P}$$

Therefore

$$\begin{aligned}
 (44) \quad t^P &= pr_p/r \\
 &= \frac{pd(d+D) + pDd_p(p-P)}{(pd+PD)(d+D)} \\
 &= \frac{(pd - e^P dD(p-P))/(d+D)}{(pd+PD)} = \frac{pd - e^P m}{(pd+PD)}
 \end{aligned}$$

Similarly,

$$\begin{aligned}
 (45) \quad T^P &= Pr_P/r \\
 &= \frac{PD + E^P m}{(pd+PD)}
 \end{aligned}$$

Substituting (21) - (24) into (20) gives

$$(46) \quad \frac{pd - me^P}{(pd - me^P)f^r/r + de^P} = \frac{PD + mE^P}{(PD + mE^P)f^r/r + DE^P} = \frac{me^a - a}{me^a f^r/r - de^a} = \frac{-mE^A - A}{-mE^A f^r/r - E^A D}$$

Inverting all expressions in (46)

$$(47) \quad \frac{f^r}{r} + \frac{de^P}{pd - me^P} = \frac{f^r}{r} + \frac{DE^P}{PD - mE^P} = \frac{f^r me^a}{r(me^a - a)} - \frac{de^a}{me^a - a} = \frac{f^r mE^A}{r(mE^A + A)} + \frac{DE^A}{mE^A + A}$$

Subtracting $\frac{f^r}{r}$ from each term in (47)

$$(48) \quad \frac{de^P}{pd - me^P} = \frac{DE^P}{PD - mE^P} = \frac{f^r a}{r(me^a - a)} - \frac{de^a}{me^a - a} - \frac{-f^r A}{r(mE^A + A)} + \frac{DE^A}{mE^A + A}$$

Inverting each expression in (48)

$$(49) \quad \frac{p}{e^P} - \frac{m}{d} = \frac{P}{E^P} + \frac{m}{D} = \frac{me^a - a}{af^r/r - de^a} = \frac{-mE^A + A}{-Af^r/r + DE^A}$$

(49) can be alternatively expressed

$$(50) \quad \frac{P}{e^P} - \frac{m}{d} = \frac{P}{E^P} + \frac{m}{D} = \frac{\frac{a}{e^a} - m}{d - \frac{a}{e^a} \cdot \frac{f^r}{r}} = \frac{\frac{A}{E^A} + m}{D - \frac{A}{E^A} \cdot \frac{f^r}{r}}$$

Expression (50) gives the first-order conditions in their final form. Optimal pricing and promotion policies in both market segments can now be determined by making the appropriate pairwise comparisons between equations in (50).

Consider the first two equations in (50) and substitute (35) and (37) into them. This gives

$$\frac{P}{e^P} + r - p = \frac{P}{E^P} + r - P$$

or

$$(51) \quad p(1 - 1/e^P) = P(1 - 1/E^P)$$

The left and right hand sides of (51) are simply the marginal revenues from sales in the first and second market segments. Equation (51) gives the optimal pricing decisions in the first and second market and the relationship between them.

Without loss of generality, assume that, at the optimum, $e^P < E^P$, so that $p > P$ and $m > 0$.

Consider the first and third equations in (50). Since they are equal

$$\left(\frac{p}{e^p} - \frac{m}{d}\right) \left(d - \frac{af^r}{e^a r}\right) = \frac{a}{e^a} - m$$

$$\frac{pd}{e^p} - m - \frac{apf^r}{e^a r e^p} + \frac{maf^r}{r d e^a} = \frac{a}{e^a} - m$$

$$\frac{pd}{e^p} \cdot \frac{e^a}{a} = 1 + \frac{f^r}{r} \left(\frac{p}{e^p} - \frac{m}{d}\right)$$

$$\begin{aligned} \frac{e^a}{a} &= \frac{e^p}{pd} + \frac{f^r}{rd} - \frac{e^p}{pd} \cdot \frac{f^r m}{rd} \\ &= \frac{e^p}{rd} + \frac{f^r}{d} + \frac{e^p}{d} \left(\frac{1}{p} - \frac{1}{r} - \frac{1}{p} \cdot \frac{f^r m}{rd}\right) \\ &= \frac{e^p}{rd} + \frac{f^r}{dr} + \frac{e^p}{dr} \left(\frac{r}{p} - 1 - \frac{f^r (p-r)}{p}\right) \end{aligned}$$

That is,

$$\frac{e^a}{a} = \frac{e^p + f^r - e^p(1 + f^r)(1 - \frac{r}{p})}{rd}$$

or

$$(52) \quad \frac{a}{rd} = \frac{e^a}{e^p + f^r - e^p(1 + f^r)(1 - \frac{r}{p})}$$

This equation gives the optimal advertising decision for the first market segment when an optimal pricing policy is pursued in that segment.

By a similar comparison of the second and fourth equations in (50), the corresponding optimal advertising decision in the second market is given by

$$(53) \quad \frac{A}{rD} = \frac{E^A}{E^P + f^r - E^P(1 + f^r)(1 - \frac{r}{p})}$$

Note that when a single market is assumed, (52) and (53) collapse to the Nerlove-Waugh theorem.

The relationship between advertising in the two market segments can be examined by considering (52) and (53). Under the convention that $e^P < E^P$, which gives $P < p$, then $P < r < p$, and

$$(54) \quad \frac{a}{rd} > \frac{e^a}{e^P + f^r}$$

and

$$(55) \quad \frac{A}{rD} < \frac{E^A}{E^P + f^r}$$

Now this implies that

$$\frac{a}{rde^a} > \frac{1}{e^P + f^r} > \frac{1}{E^P + f^r} > \frac{A}{rDE^A}$$

or

$$(56) \quad \frac{a/d}{A/D} > \frac{e^a}{E^A}$$

That is, the ratio of advertising per unit sales in the less price elastic market segment to that in the more price elastic market segment exceeds the ratio of the corresponding advertising elasticities.

CHAPTER 4

CONCLUSIONS

In the previous Chapter, optimal pricing and promotion rules were determined for an agricultural marketing agency which has control over these marketing variables, but not over production. These optimal policies will now be briefly compared with those prescribed by alternative models.

The optimal pricing policy was to set prices in the two market segments so as to equate the marginal revenue from selling in each market. That is, a higher price should be charged in the market segment with the lower price elasticity. Thus, the conventional rule of the price-discriminating monopolist for allocating output to market segments should be maintained, even though the marketing agency is required to sell all output supplied to it. However, the marginal revenues in these individual market segments are not required to equal the marginal cost of production, and hence, the profit-maximising monopoly level of output is not produced.

Given optimal pricing, the optimal ratio of advertising in a market segment to producer returns from that segment is given by equations (52) and (53). That is, optimal advertising in a segment is a function of the advertising and price elasticities of demand in that segment, the price elasticity of supply, and a measure of the relationship between the optimal price in that segment and the pool price returned to suppliers.

Recall the Nerlove-Waugh theorem that a marketing agency facing a single market should choose policies such that the ratio of advertising to sales revenue equals the ratio of the advertising elasticity to the sum of the demand

and supply elasticities. By comparison, a price-discriminating marketing agency should choose advertising policies such that in the market segment with the lower (higher) price elasticity of demand, the ratio of advertising to producer payments should exceed (be less than) the ratio of the advertising elasticity in that market to the sum of the demand and supply elasticities in that market. That is, relatively more (less) advertising effort (as measured by the advertising to producer returns ratio) would be directed to the less (more) price elastic market segment than would be the case if this was the only market faced by the agency.

Finally, consider relative advertising effort in each market segment, and recall from inequality (56) that the ratio of advertising per unit sales in the less price elastic market to that in the more price elastic market exceeds the ratio of the corresponding advertising elasticities. Equation (23) indicates that where pricing is uniform across market segments and market determined, then the ratio of advertising per unit sales in one market segment to that in the other market segment equals the ratio of the corresponding advertising elasticities. That is, under optimal pricing, relatively more (less) advertising effort (as measured by advertising per unit sales) is directed to the less (more) price elastic market segment than under uniform pricing across these segments. This result makes intuitive sense, since relatively more advertising effort is directed to the less price elastic segment where the potential to exploit monopoly power through discriminatory pricing is greater.

REFERENCES

- De Boer, A.J. (1977), Rural Product Promotion: Economic Aspects of Promotability, Organisation and Public Assistance. Review of Marketing and Agricultural Economics, 45(4): 121-145.
- Dorfman, R. and Steiner, P.O. (1954), Optimal Advertising and Optimal Quality. American Economic Review, 44(2): 826-836.
- Kotler, P. (1984), Marketing Management (5th Ed.). New Jersey, Prentice-Hall, 792 p.
- Lambin, J.J. (1976), Advertising, Competition and Market Conduct in Oligopoly over Time. Amsterdam, North-Holland, 312 p.
- Martin, S.K. (1985), An Extension of the Nerlove-Waugh Theorem to Multiple Market Segments. Unpublished Working Paper, Department of Agricultural Economics and Marketing, Lincoln College.
- Nerlove, M. and Waugh, F.V. (1961), Advertising without Supply Control: Some Implications of a Study of the Advertising of Oranges. Journal of Farm Economics 43(4): 813-837.
- Nerlove, M. and Arrow, K.J. (1962), Optimal Advertising Policy under Dynamic Conditions. Economica 29 (May 1962): 129-142.
- Strak, J., Optimal Advertising Decisions for Farmers and Food Processors. Journal of Agricultural Economics 34(3): 303-315.